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KINETIC MODELING OF FED-BATCH BIO-REACTOR VOLUME FOR THE TREATMENT OF INDUSTRIAL WASTES

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ABSTRACT

A Fed-Batch Bio-reactor volume changes as the industrial wastes (feed) is put into the reactor. This change in volume is a function of time and is equal to the rate at which the wastes is fed into the reactor. The reactor has to be filled to this volume before the outlet valve could be opened. The volume of the bio-reactor is therefore the volume of the content of the reactor and hence it could be found using materials balance equation. To find this volume, four kinetic models were derived, one for the concentration of Biomass, one for the concentration of industrial wastes, one for the volume of industrial wastes to be filled into the bio-reactor and one for the volume of the bio-reactor cylindrical space. These kinetic models solved with the help of a MATLAB program gave the volume of industrial wastes to be filled into the bio-reactor, the volume of the bio-reactor cylindrical space, the concentration of biomass and the concentration of the industrial wastes in the bio-reactor for a Hydraulic Retention Time (HRT) of five days, hydraulic retention time being the time taken to attain the Nigerian Federal Environmental Protection Agency (FEPA) discharge limits. The volume of the bio-reactor on the day Nigerian FEPA discharge limits were attained was taken as the volume of the bio-reactor. This gave 25.0 m^3 *for volume of industrial wastes to be filled into the bio-reactor and 32.35* m^3 *for volume of the bio-reactor cylindrical space. The kinetic models were validated using Root Mean Square Error (RMSE). A Root Mean Square Error of 0.85* shows *that 85% of the variability in the data set was explained by the kinetic models.*

KEYWORDS:
Fed-Batch; Bio-reactor; Kinetic model; Rate of reaction; Volumetric feed-rate; Industrial Wastes.

1.0. INTRODUCTION

Sampson (2018) defined a bio-reactor as a vessel in which processes occur through which micro-organisms breakdown harmful biodegradable material to harmless ones thereby synthesizing useful products.

Industrial wastes are the undesired product of any industrial activity or unit operation. These wastes which are usually toxic and hazardous causing diseases need to be treated in bio-reactors so as to make them non-toxic to personnel on exposure and harmless to the environment on disposal.

The design of a bio-reactor imposes a major challenge to engineers and scientists. In Nigeria, the treatment and disposal of industrial wastes to FEPA standard and regulations imposes a major challenge to the oil and gas industry. On the basis of mode of operation, bio-reactors are classified as continuous, plug flow, batch and semi-continuous or semi-batch (fed-batch or batch-fed). An example of a continuous bio-reactor is the chemostat.

Rao (2010) defined a batch bio-reactor as a bio-reactor fed with all reactants and sealed with no further inflow or outflow of materials, the products withdrawn after the reaction time is over.

Continuous stirred tank bio-reactors are mixed flow bio-reactors, the flow rate of the feed is equal to the flow rate of the product out of the reactor with the contents well mixed with the help of a stirrer. In plug-flow bio-reactors, there is no mixing of reactants but conversion occurs as the reactants move from one end of the reactor to the other.

Semi-continuous or semi-batch bio-reactors are grouped into two: the Batch-fed and the Fed-Batch bio-reactors. Tapobrata (2011) defined a Batch-fed bio-reactor as a bio-reactor initially started as variable volume with products withdrawn when it is formed and the feed injected only when some product is removed.

Tapobrata (2011) defined a Fed-batch bio-reactor as a bio-reactor initially started as variable volume batch with no output till the desired volume is achieved. It is then switched on to continuous flow mode but the input feed rate and output flow rates are not constant. These feed rates depend on an appropriate metabolic function, the respiratory Quotient of the process organism.

Wikipedia (2019) defined Chemical Kinetics as the study of reaction rates, the change in concentrations of reactants and products with time.

A model is a representation often mathematical of a process, concept or operation of a system. A kinetic model is a mathematical representation of a process, concept or operation of a system showing reaction rates, the change in concentrations of reactants and products with time. A kinetic model could be defined as an equation that shows the rate at which certain parameters in a system do change with time. Example of such parameters may be concentration, pressure or volume, etc. For a fed-batch bio-reactor, the volume of the reactor changes with time or say the volume is a function of time. This change in volume is dependent on the feed rate of the industrial wastes into the reactor.

Rao (2010) gave a generalized model for a fed-batch bio-reactor as below:

$$\frac{dC_i}{dt} = \frac{V\,(t)}{V_R}\left(C_{i,0} - C_i\right) + rf_i \qquad (1)$$

Where V(t) is the volumetric feed rate at time t.
V_R, the reactor volume which is a function of time t.
Rf_i, the rate of reaction of component i
if there is a suitable analytical expression for rf_i, equation (1) can be integrated.
$C_{i,0}$ the initial concentration of component i
C_i, the final concentration of component i.

The volume of anaerobic reactor can be calculated using a relationship in Tchobanoglous *et al.* (2004):

$$V_R = q \times t \tag{2}$$

Where: q is the volumetric feed rate and t the hydraulic retention time. Equation (2) is similar to the kinetic model for finding the volume of the anaerobic reactor industrial wastes:

$$V_R\ (t)\ = V.t + 0.0015 \tag{3}$$

Only that the constant of integration 0.0015 m^3 is neglected hence reducing its precision.

2.0.METHODS

2.1. Development of Kinetic Model for finding the concentration of Biomass in the Bio - Reactor.

$$r_A \propto C_A$$

$$r_A = KC_A \tag{4}$$

$$r_A V_R = KC_A V_R \tag{5}$$

$$KC_A V_R = \mu C_1 V_R \tag{6}$$

The materials balance equation:
[Accumulation] = [Flow in] + [Microbial Biodegradability] – [Flow out]

$$V_R \frac{dC1}{dt} = VC_{1,0} + \mu C_1 V_R - VC_1 \tag{7}$$

Dividing through by V_R

$$\frac{V_R dC_1}{V_R dt} = \frac{V}{V_R} C_{1,0} + \frac{\mu C_1}{V_R} - \frac{VC_1}{V_R}$$

$$\frac{\cancel{V_R} dC_1}{\cancel{V_R} dt} = \frac{V}{V_R} C_{1,0} + \frac{\mu C_1 \cancel{V_R}}{\cancel{V_R}} - \frac{VC_1}{V_R}$$

This can be re-written as:

$$\frac{dC_1}{dt} = \frac{V}{V_R} C_{1,0} + \mu C_i - \frac{VC_1}{V_R} \tag{8}$$

Taking $\frac{V}{V_R}$ on the right hand side out because it is common:

$$\frac{dC_1}{dt} = \frac{V}{V_R} (C_{1,,0} + C_1) + \mu C_1 \tag{9}$$

Where V is the volumetric feed rate and V_R, the volume of the bio-reactor
Let $\frac{V}{V_R} = D$, the dilution rate or the space velocity.
Substitute $\frac{V}{V_R}$ as D

$$\frac{dC_i}{dt} = D(C_{1,0} - C_i) + \mu C_1 \tag{10}$$

From Monods equation:

$$\mu = \frac{\mu_m C_2}{K_m + C_2}$$

Substitute this Monods equation for μ in equation (10)

$$\frac{dC_1}{dt} = D(C_{1,0}\text{-}C_1) + \frac{\mu_m C_1 C_2}{K_m + C_2} \qquad (11)$$

Writing equation (10) in the pattern of the modified form of Monods equation to account for the consumption of cellular material to produce maintenance energy. The modified form of Monods equation:

$$\mu = \frac{\mu_m C_2}{K_m + C_2} - k_d \qquad (12)$$

Substituting $\mu = \frac{\mu_m C_2}{K_m + C_2} - k_d$ into equation (11)

$$\frac{dC_1}{dt} = D(C_{1,0}\text{-}C_1) + \frac{\mu_m C_1 C_2}{K_m + C_2} - k_d C_1 \qquad (13)$$

This can be written as:

$$\frac{dC_1}{dt} = \frac{V}{V_R}(C_{1,0}\text{-}C_1) + \frac{\mu_m C_1 C_2}{K_m + C_2} - k_d C_1$$

Considering $\frac{V}{V_R} = D$

Equation (13) resembles an equation in Coulson & Richardson (1991):

$$\frac{dx_j}{dt} = -Dx_j + \frac{\mu_{mj}\, Sx_j}{K_m + S} - \frac{\mu m_k X_j X_k}{Y_k\ (KS_k + Y_k)} \qquad (14)$$

Where x_j is the final concentration of the biomass.

μ_{mj}, maximum specific Growth rate or half the maximal velocity concentration for component j

S, Final substrate (Industrial wastes) concentration

μ_{mk}, maximum specific Growth rate or half the maximal velocity concentration for component k

x_k, Final concentration of Biomass

Y_k, Yield coefficient

KS_k, Monods constant or substrate saturation constant

D, the dilution rate, a ratio of the volumetric feed rate to the culture volume of the bio reactor.

t, the time.

2.2. Development of Kinetic Model for finding the Final Concentration of the Industrial Wastes

$$r_A \propto C_A$$

$$r_A = KC_A$$

$$RC_i = \mu C_A \qquad (15)$$

$$KC_A V_R = \mu C_1 V_R \qquad (16)$$

The Materials balance equation:

[Accumulation] = [Flow in] + [Microbial Biodegradability] – [Flow out]

$$\frac{V_R dC_2}{dt} = V.C_{2,0} - \frac{\mu C_1 V_R}{Y} - V.C_2 \qquad (17)$$

Rearranging:

$$V_R \frac{dC_2}{dt} = V.(C_{2,0} - C_2) - \frac{\mu C_1 V_R}{Y} \qquad (18)$$

Bringing out V, being a common term on the right hand side:

$$\frac{V_R dC_2}{dt} = VC_{2,0} - V.C_2 - \frac{\mu C_1 V_R}{Y} \qquad (19)$$

Dividing through by V_R:

$$\frac{V_R dC_2}{V_R dt} = \frac{V}{V_R}(C_{2,0} - C_2) - \frac{\mu C_1 V_R}{Y V_R} \qquad (20)$$

$$\frac{\cancel{V_R} dC_2}{\cancel{V_R} dt} = \frac{V}{V_R}(C_{2,0} - C_2) - \frac{\mu C_1 \cancel{V_R}}{Y \cancel{V_R}}$$

$$\frac{dC_2}{dt} = \frac{V}{V_R}(C_{2,0} - C_2) - \frac{\mu C_1}{Y} \qquad (21)$$

Substituting the Monods equation

$\mu = \frac{\mu_m C_2}{K_m + C_2}$ into equation (21)

$$\frac{dC_2}{dt} = \frac{V}{V_R}(C_{2,0} - C_2) - \frac{\mu_m C_1 C_2}{Y\ (K_m + C_2)} \qquad (22)$$

Equation (22) resembles an equation in coulson & Richardson (1991):

$$\frac{dS}{dt} = D(S_0 - S) - \frac{\mu_{mj}\, S X_j}{Y_j\ (K_{ij} + S)} \qquad (23)$$

Where

S is the Final Substrate (Industrial wastes) concentration
t, the time
D, the dilution rate or space velocity
S_0, Initial substrate (Industrial wastes) concentration
S, Final substrate (Industrial wastes) concentration
μ_{mj} , maximum specific Growth rate or half the maximal velocity concentration
X_j , Final Concentration of Biomass
Y_j , Yield coefficient
K_{ij} , the Monods constant or substrate saturation constant.

2.3. (a) Development of Kinetic Model for finding the Volume of Industrial Wastes to be filled into the Fed-batch Bio-reactor (V_R)

Since the reactor is being filled, the volume, V varies with time. The reactor volume at any time t can be found from an overall mass balance of all species (Fogler, 2006)

The mass balance equation:

[Rate in] = [Rate Out] + [Rate of Generation] = [Rate of Accumulation] (24)

$\rho_0 V$ o 0 $\frac{d\rho V_R}{d_t}$

The outlet valve is closed to enable the reactor to be filled. There is no reaction yet while filling the reactor because it is after filling the reactor that bacteria is added to the industrial wastes in the reactor. So for now, there is no generation or generation = 0

$$[\text{Rate of Accumulation}] = [\text{Rate in}] \tag{25}$$

$$\frac{d\rho V_R}{dt} = \rho_0 V \tag{26}$$

Where ρ is the final density,
ρ_0 the initial density,

$$V_R = Area \times height = Ah \tag{27}$$

Putting Ah for V_R

$$\frac{d\rho A\,h}{dt} = \rho_o V \tag{28}$$

$$\frac{\rho Ad\,h}{dt} = \rho_o V \tag{29}$$

Quasi-Steady State:

Since ρ is always approximately equal to ρ_o at different times of measurement, ρ_o can be substituted for ρ.

$$\frac{\rho_o Adh}{dt} = \rho_o V \tag{30}$$

Dividing both sides by ρ_o

$$\frac{\cancel{\rho_o}Adh}{\cancel{\rho_o}dt} = \frac{\cancel{\rho_o}V}{\cancel{\rho_o}} \tag{31}$$

Dividing both sides by A

$$\frac{\cancel{A}dh}{\cancel{A}dt} = \frac{V}{A} \tag{32}$$

$$\frac{dh}{dt} = \frac{V}{A} \tag{33}$$

$$\frac{V}{A} = V_e \tag{34}$$

Where V_e is the velocity of the industrial wastes

$$\frac{dh}{dt} = V_e \tag{35}$$

Multiplying both sides by A

$$A\frac{dh}{dt} = V_e \times A \tag{36}$$

$$\frac{dV_R}{dt} = V_e \times A \tag{37}$$

Equation (37) be solved using MATLAB software program.

Recall $\frac{dh}{dt} = V_e$

$$\int \frac{dh}{dt} dt = \int V_e \; dt \tag{38}$$

$$h\,(t) = V_e \int dt \tag{39}$$

$$h\,(t) = V_e . t + C \tag{40}$$

Multiplying both sides by A

$$Ah\,(t) = V_e . t\,. A + C \tag{41}$$

$$Ah\,(t) = V_R(t) \tag{42}$$

Hence Ah (t) is replaced with V_R (t)

The General Solution:

$$V_R(t) = V_e\,.\,t\,.A + C \tag{43}$$

To find C, substitute initial conditions at t = 0

$$V_R(0) = V_e \times 0 \times A + C$$

$$V_R(0) = 0 + C \tag{44}$$

$$V_R(0) = C \tag{45}$$

The particular solution:

$$V_R(t) = V_e \times t \times A + 0.0015 \qquad (46)$$

Dimensional Analysis:

$$V_R(t) = V_e \times t \times A + C$$

$$\frac{m}{\cancel{d}} \times \cancel{d} \times m^2 = m^3$$

Where C is a constant of integration with unit m^3

At t = 0

$$V_R(0) = 0.0015 \text{ m}^3$$

At t = 1:

$$V_R(1) = V_e \times 1 \times A + C \qquad (47)$$

At t = 2:

$$V_R(2) = V_e \times 2 \times A + C \qquad (48)$$

At t = 3:

$$V_R(3) = V_e \times 3 \times A + C \qquad (49)$$

At t = 4:

$$V_R(4) = V_e \times 4 \times A + C \qquad (50)$$

At t = 5:

$$V_R(5) = V_e \times 5 \times A + C \qquad (51)$$

From laboratory experimental results, Nigerian FEPA discharge limits were reached on day 5, hence the volume of wastewater to be filled into the reactor is taken as the volume on day 5 = 25.0 m^3

Recall:

$$\frac{dV_R}{dt} = V_e \times A$$

$$\text{And} \quad V_e \times A = V$$

Hence,

$$\frac{dV_R}{dt} = V \qquad (52)$$

Where V_R is the volume of the bio-reactor industrial wastes, t is the time and V, the Volumetric feed rate.

2.3. (b) Development of Kinetic Model for finding the volume of the Fed-Batch Bio-reactor cylindrical space (V_r Cylindrical Space)

$$[Ratein] - [RateOut] + [RateofGeneration] = [RateofAccumulation] \qquad (53)$$

$$\rho_o V - o \quad + \quad \frac{\mu_m c_1 c_2}{Y\ (k_m + c_2)} = \frac{d\rho V_r\ (Cylindrical\ \ Space\,)}{dt} \qquad (54)$$

Substituting Ah for V_r Cylindrical Space:

$$\rho_o V \quad + \quad \frac{\mu_m c_1 c_2}{Y\ (k_m + c_2)} = \frac{d\rho A h}{dt} \qquad (55)$$

$$\rho_o V \quad + \quad \frac{\mu_m c_1 c_2}{Y\ (k_m + c_2)} = \frac{\rho A d h}{dt} \qquad (56)$$

Quasi – steady state condition:
Considering that ρ is always approximately equal to ρ_o at different times of measurement. ρ_o can be substituted with ρ.

$$\rho V \quad + \quad \frac{\mu_m c_1 c_2}{Y\,(k_m + c_2)} = \frac{\rho A d\,h}{dt} \tag{57}$$

Dividing both sides of equation (57) by ρ

$$\frac{\cancel{\rho}V}{\cancel{\rho}} + \frac{\mu_m c_1 c_2}{Y\,(k_m + c_2)} = \frac{\cancel{\rho}Adh}{\cancel{\rho}dt}$$

$$V \; + \; \frac{\mu_m c_1 c_2}{Y\,(k_m + c_2)} = \frac{Ad\,h}{dt} \tag{58}$$

$$V_e\,.A \; + \; \frac{\mu_m c_1 c_2}{Y\,(k_m + c_2)} = \frac{Ad\,h}{dt} \tag{59}$$

Rewriting:

$$\frac{Ad\,h}{dt} = V_e\,.A \; + \; rf_i \tag{60}$$

$$\frac{dV_r\,cylindricalspace}{dt} = V_e\,.A \; + \; rf_i \tag{61}$$

Equation (61) can be solved using a suitable MATLAB software program.

Recall

$$V + \frac{\mu_m c_1 c_2}{Y\,(k_m + c_2)} = \frac{Adh}{dt}$$

Dividing both sides by A

$$\frac{V}{A} + \frac{\mu_m c_1 c_2}{Y\,(k_m + c_2)} = \frac{\cancel{A}dh}{\cancel{A}dt}$$

$$V_e\,.A = V \tag{62}$$

Hence, $V_e = \frac{V}{A}$

Substituting $V_e \;\; for \frac{V}{A}$ in equation (62)

$$V_e + \frac{\mu_m c_1 c_2}{Y\,(k_m + c_2)} = \frac{dh}{dt} \tag{63}$$

Integrating both sides of equation (63) with respect to time.

$$\int (V_e) dt + \int \frac{\mu_m c_1 c_2}{Y\,(k_m + c_2)} dt = \int \left(\frac{dh}{dt}\right) dt \tag{64}$$

$$V_e.t + \frac{\mu_m c_1 c_2}{Y\,(k_m + c_2)} t + c_a = h(t) \tag{65a}$$

Let $\frac{\mu_m c_1 c_2}{Y\,(k_m + c_2)} = rf_i$

Rewriting equation (65a)

$$h(t) = V_e . t + trf_i + Ca \tag{65b}$$

where c_a is a constant of integration.

Since Volume = Area × height

Let $V_r CylindricalSpace$= A × h(t) (66)

Dividing both sides by A

$$h(t) = \frac{V_r CylindricalSpace}{A} \tag{67}$$

Substituting $\frac{V_r\ CylindricalSpace}{A}$ for h (t) in equation(65b)

$$\frac{V_r\ CylindricalSpace}{A} = V_e . t + trf_i + c_a \tag{68}$$

Note that the rate of reaction is only added to account for its effects on the change in volume of the reactor cylindrical space (Sampson, 2021a)

$$\frac{V_r\ Cylindrical\ Space}{A} \times A = A . V_e . t + Ac_a + trf_i$$

The General Solution:

$$V_r\ CylindricalSpace = A . V_e . t + Ac_a + trf_i$$

Equation (69) is called 'Idongesit Effiong Sampson equation being the first with addition of reaction rate.

$$V_r\ Cylindrical\ Space(t) = A . V_e . t + Ac_a + trf_i \tag{69}$$

At t = 0, $rf_i = 0$

$$V_r\ Cylindrical\ Space(0) = A . V_e . (0) + Ac_a + (0)(0)$$

$$V_r\ Cylindrical\ Space(0) = 0 + Ac_a + 0$$

$$V_r\ Cylindrical\ Space(0) = Ac_a$$

Let $Ac_a = C$

Hence,

$$V_r\ Cylindrical\ Space(0) = C$$

Where C is a constant of integration equal to the volume at initial conditions

t = 0

This is the volume of the anaerobic jar used for laboratory experiment = 1.5 Litres = 0.0015 m^3

Hence,

$$V_r\ CylindricalSpace(0) = 0.0015\ m^3$$

Substituting 0.0015 for C_a in the general solution to obtain the particular solution

The particular solution:

$$V_r\, Cylindrical\, Space(t) = V_e \,.\, t \,.\, A + trf_i + 0.0015\, A \qquad (70)$$

$$V_r\, Cylindrical\, Space(t) = V_e \,.\, A.t + rf_i \,.\, t + 0.0015\, A \qquad (71)$$

$$V_r\, Cylindrical\, Space(t) = [V_e \,.\, A + rf_i\,]\, t + 0.0015\, A \qquad (72)$$

$$V_r\, Cylindrical\, Space(t) = [V_e \,.\, A + rf_i\,]\, t + 0.0015 \times 3.80182$$

$$V_r\, Cylindrical\, Space(t) = [V_e \,.\, A + rf_i\,]\, t + 0.00570273$$

$$V_r\, Cylindrical\, Space(t) = [(1.315159581)(3.80182) + 1.468]t + 0.00570273$$

$$V_r\, Cylindrical\, Space(t) = 6.467999998t + 0.00570273$$

The particular solution is:

$$V_r\, CylindricalSpace(t) = 6.467999998t + 0.00570273 \qquad (73)$$

Substituting the values of t into this particular solution gives the volume of the bio-reactor cylindrical space ($V_r\ cylindrical\ space$) which is a function of time.

Rao (2010) stated that the volume of a fed-batch mode operated bio-reactor is a function of time.

At day 5, Nigerian FEPA discharge limits were attained, so volume at day 5, is taken as the volume of the bio-reactor cylindrical space ($V_r\ Cylindrical\ Space$)**= 32.35** m^3
25.0 m^3 of industrial wastes was treated every 5 days in a bio-reactor of volume 32.35 m^3.

Dimensional Analysis:

$$V_r Cylindrical\, Space(t) = V_e \times t \times A + C + trf_i$$

$$\frac{m}{\cancel{d}} \times \cancel{d} \times m^2 + Constant\ of\ integration$$
$$+ a\ constant\ dimensionless\ value\ which\ is\ a\ function\ of\ time$$

m^3 + Constant of integration + a dimensionless value

The main side is m^3

No Unit of measurement contains a plus sign or a minus sign, so the Quantities subtracted or added cannot affect the Unit of the Main side which is the Unit of the Quantity of interest which is the Unit of Volume.

The Rate of reaction does not change the Unit of Volume which is meter cube.

The unit of measurement of volume of a bio- reactor remains metre cube even when reaction is taking place in the bio-reactor.

3.0.RESULTS AND DISCUSSION

3.1.Volume of the Fed-Batch Bio-reactor (FBB)

Four Kinetic Models:

i. Equ (13) $$\frac{dC_1}{dt} = \frac{V}{V_R}\left(C_{1,0} - C_1\right) + \frac{\mu_m C_1 C_2}{k_m + C_2} - kdC_1$$

ii. Equ (22) $$\frac{dC_2}{dt} = \frac{V}{V_R}\left(C_{2,0} - C_2\right) - \frac{\mu_m C_1 C_2}{\gamma(k_m + C_2)}$$

iii. Equ (37) $$\frac{dV_R}{dt} = V_e \times A$$

iv. Equ (61) $$\frac{dV_{r\ cylindrical\quad space}}{dt} = V_e \times A + rf_i$$

Solved with MATLAB software program in Fourth order Rungekutta yielded a volume of bio-reactor industrial wastes V_R= 25 m^{3}and a volume of bio-reactor cylindrical space (V_r Cylindrical space) = 32.35 m^3

Table 1: Volume of Bio-Reactor, Concentration of Industrial Wastes and Concentration of Biomass using MATLAB Software Program in Fourth Order Rungekutta

Time (t) (days)	Volume of Industrial Wastes filled into the Bio-Reactor (V_R) m^3	Volume of Bio-Reactor Cylindrical Space (V_r) m^3	Concentration of Industrial Wastes (C_2) (mg/L)	Concentration of Biomass (C_1) (mg/L)
0	0.0015	0.0015	1.000	0.000015
1	5.0015	6.4737	0.0014	0.00141
2	10.0015	12.9417	0.0008	0.00135
3	15.0015	19.4097	0.0005	0.00130
4	20.0015	25.8777	0.0004	0.00125
5	25.0015	32.3457	0.0003	0.00121

FEPA discharge limit for Fed-batch bio-reactor was reached at day (5). The highest volume given by the MATLAB program was taken as the volume of the bio- reactor industrial wastes (V_R = 25.0015 m^3). A volume of bio-reactor cylindrical space of 32.35 m^3 show that industrial wastes of volume 25.0 m^3 was treated every 5 days in a bio-reactor of volume 32.35 m^3.

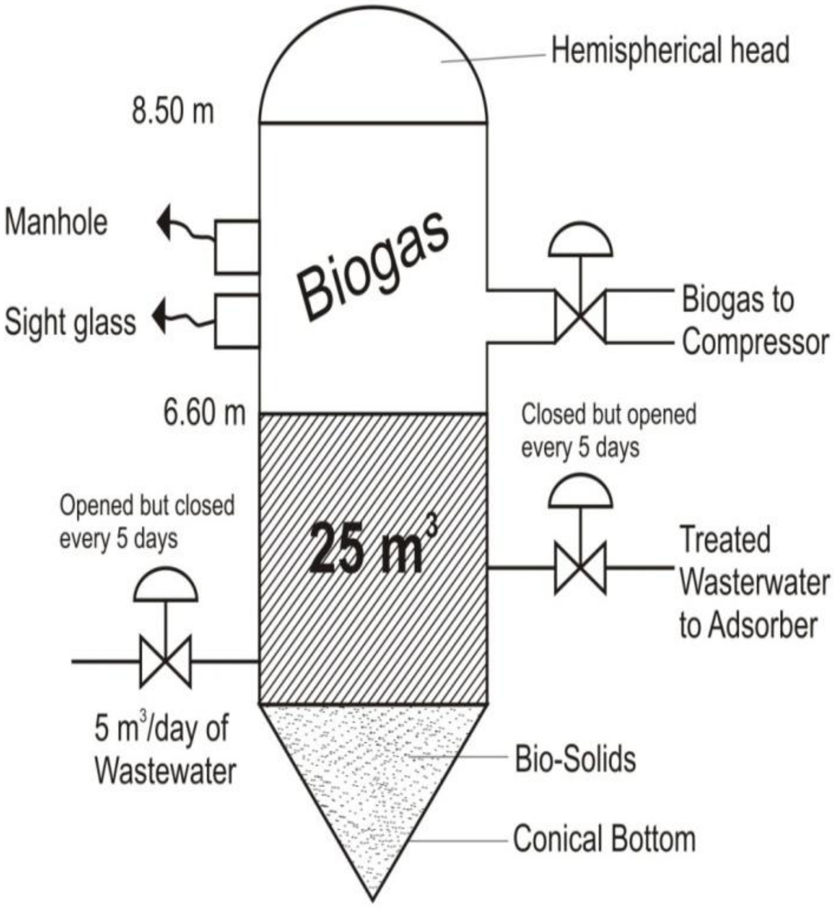


Fig. 1: Fed – Batch Mode for Operation of the Bio-Reactor for a Case where Reaction rate was added to the Volume of the bio-reactor as biogas is to stay in the bio-reactor

The inlet valve was always kept open and closed after five days before it was opened again. The outlet valve was closed until after 5 days before it was opened for part of the treated industrial wastes to flow out after which it was closed.

With this, any industrial wastes that flowed into the bio-reactor stayed for at least five days before it flowed out.

Therefore volume of the bio-reactor industrial wastes

$$= 5\ \frac{m^3}{\cancel{day}} \times 5\ \cancel{days}$$

$$= 25\ m^3$$

25.0 m^3 of industrial waste was treated every 5 days in a fed-batch bio-reactor of volume 32.35 m^3.

3.1.1. A Case where Bio-Gas is not to stay in the Bio-reactor

The volume of industrial wastes to be filled into the bio-reactor can be taken as the volume of the bio-reactor where bio-gas is not to co-inhabit the bio-reactor with the industrial wastes. In this case, the biogas is piped out directly from the top of the bio-reactor. In this case, there is no need for a factor of safety, as there is no risk of pressure build up in the bio-reactor and there is no addition of a constant representing the effect of reaction rate to the volume of the bio-reactor. The case where bio-gas is not to stay in the bio-reactor has been found suitable for every mode of operation: batch, fed-batch or batch-fed and plug-flow.

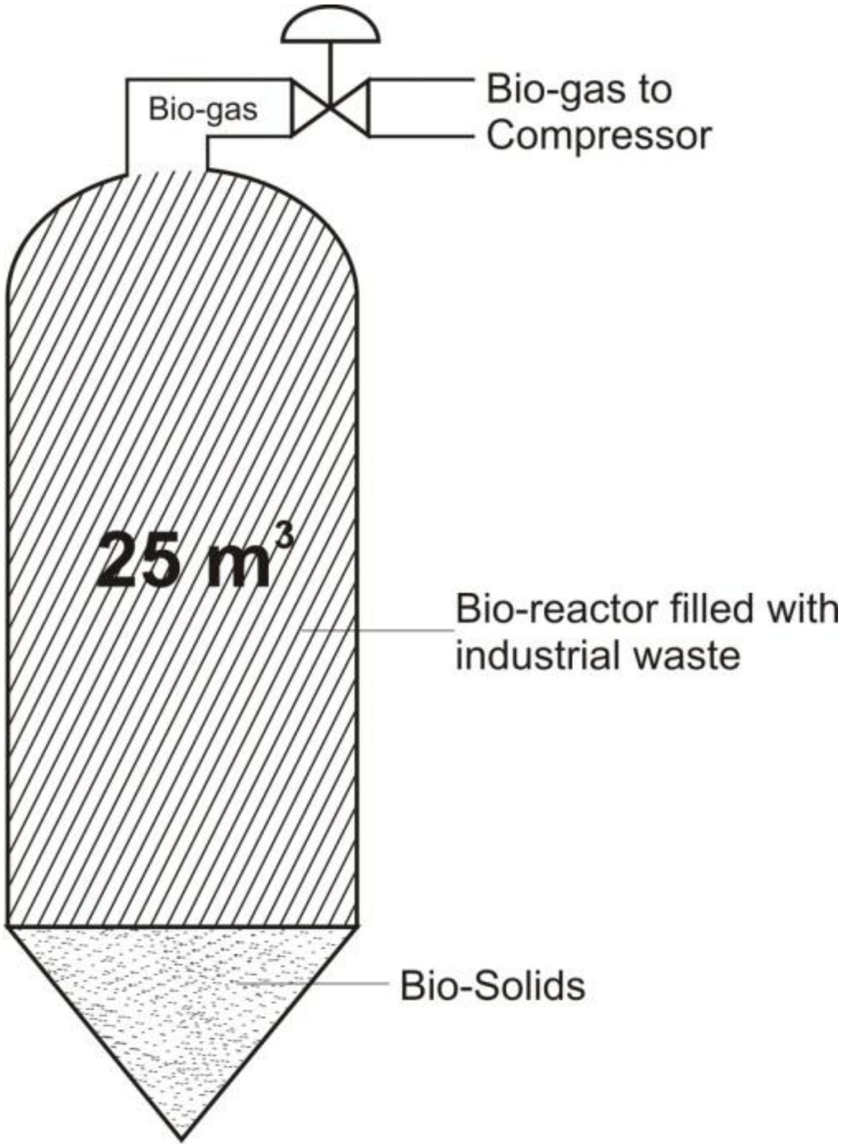


Fig 2: A Case where no Reaction Rate is added to the Volume of the bio-reactor as biogas is not to stay in the bio-reactor.

In this case, $\frac{dV_R}{dt} = V$

Integrated to yield: VR (t) = V . t + C

gives for the volume of the bio-reactor

Where V_R is the Volume of the Fed-batch bio-reactor.

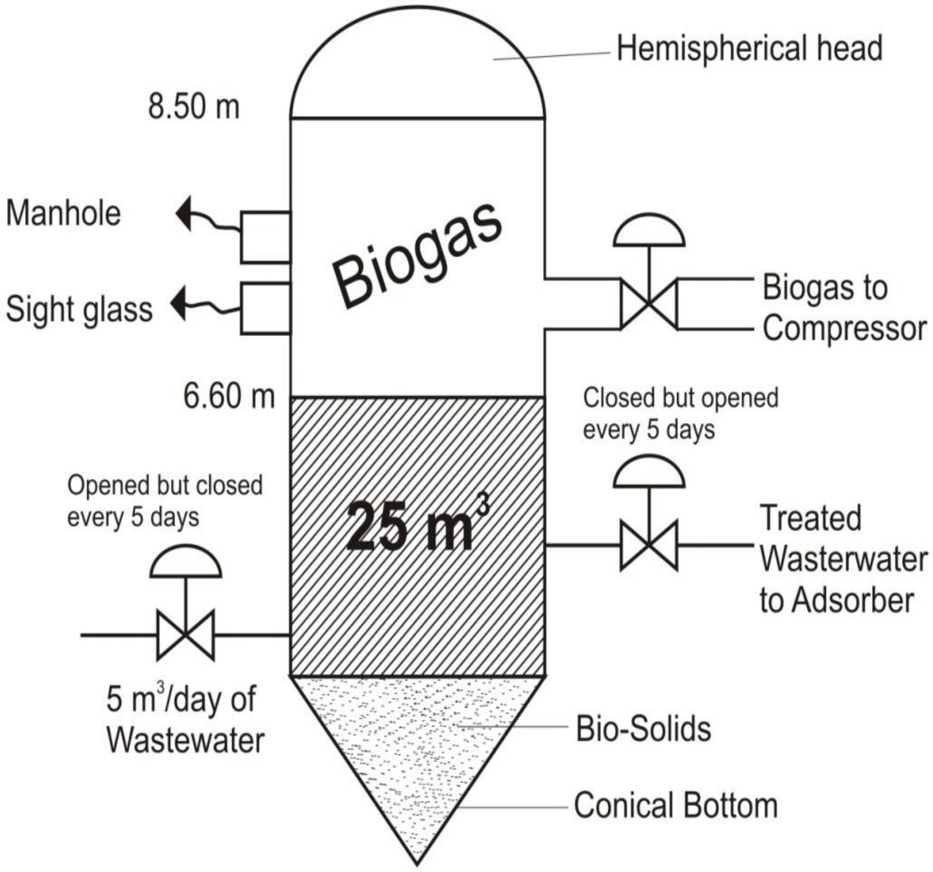
V, the Volumetric feed rate of the industrial wastes (feed) t, the time in days and C, a constant of integration (the volume of the anaerobic jar used for laboratory experiment).

3.1.2. A Case where Reaction Rate was added to the Volume of the Bio-Reactor as Bio-Gas is to stay in the Bio-Reactor

In this case, bio-gas can only be piped to the compressor after building enough pressure in the bio-reactor, hence there is need for a factor of safety as there is risk of pressure build-up in the bio-reactor.

In this case, a constant representing the effect of reaction rate is added to the volume of the bio-reactor.

This is illustrated in Figure 1. Recall Figure 1.



In this case, the following equations hold:

$$\frac{dV_r \; Cylindrical \; Space}{dt} = \mathrm{V} + \mathrm{rf_i}$$

Considering that $V_e \times A = V$

$$\frac{dV_r \; Cylindrical \; Space}{dt} = \mathrm{V_e} \times \mathrm{A} + \mathrm{rf_i}$$

This is integrated to yield:

$$V_r \; Cylindrical \; Space \; (t) = \mathrm{V_e} \times \mathrm{t} \times \mathrm{A} + \mathrm{C} + \mathrm{trf_i}$$

This is the "Idongesit Effiong Sampson equation"

This is only suitable for Fed-batch process, or batch-fed. If it were a batch process biogas would have gone with the treated wastes when all treated wastes are to be discharged to enable the treatment of fresh feed. For Fed-batch, some of the product is discharged and some of the feed added.

3.2.Validation of Kinetic Models

3.2.1. Root Mean Square Error for the Concentration of Industrial Wastes

Table 2: Root Mean Square Error for the Concentration of Industrial Wastes

Day	Concentration by Experiment, C_2, Exp	Concentration by model, C_2, model	C_2, Ep – C_2 model	[C_2 Exp - C_2 Model]^2
0	1.0	1.0	0	0
1	0.98	0.00144	0.97856	0.957579
2	0.95	0.00075	0.94925	0.90107556
3	0.85	0.00052	0.84948	0.7216162704
4	0.75	0.000406	0.749594	0.5618911646
5	0.70	0.000337	0.699663	0.4895283136
$\sum(C_2, Exp - C_2, model)^2$				**3.631690309**

The concentration of industrial wastes was measured in the laboratory and also obtained from three kinetic model equations ran on a MATLAB software program. The results obtained from the laboratory were insignificantly different from the results obtained from the three kinetic models.

The treatment of industrial wastes which is the objective function of the research has to do with the concentration of the industrial wastes, hence the importance of this model validation.

A Root Mean Square error of 0.85 shows that 85 % of variability in the data set was explained by the kinetic models. Wikipedia (2019) defined Root Mean Square Error (RMSE) as an indication of absolute fit of a model to the data. That is how close the observed data points are to the models predicted values. RMSE is an absolute measure of fit. RMSE is the square root of the variance or standard deviation of the prediction errors.

3.2.2. Root Mean Square Error for the concentration of Biomass

Table 3: Root Mean Square Error for the Concentration of Biomass

Concentration by Experiment, C_1, Exp (mg/L)	Concentration by model, C_1, model (mg/L)	C_1, Ep – C_1 model (mg/L)	$[C_1 \text{ Exp} - C_1 \text{ Model}]^2$ (mg/L)
0.000015	0.00015	-0.000135	0.000000018225
0.4	0.0014	0.3986	0.15888196
0.7	0.00135	0.69865	0.4881118225
0.9	0.0013	0.8987	0.80766169
0.99	0.00125	0.98875	0.9776265625
0.000013	0.0012	-0.001187	0.0000014090

A Root Mean Square error of 0.70 shows that 70 % of variability in the data set was explained by the kinetic models.

3.3.Bio-reactor Diameter:

The diameter of the bio-reactor was obtained in Sampson (2021b) using Optimisation Method. This gave 2.20 metres.

3.4. Results Obtained

Table 4: Results obtained

Parameter	Value
Height of Industrial wastes	6.60 m
Height of Bio-reactor	8.50 m
Diameter of Bio-reactor	2.20 m
Volume of Bio-reactor Industrial wastes	25.0 m^3
Volume of Bio-reactor Cylindrical Space	32.35 m^3
Final Concentration of Industrial Wastes	0.7 mg/L
Initial Concentration of Industrial Wastes	1.0 mg/L
Concentration of Biomass	0.000013 mg/L
Yield Coefficient	0.00016
Coefficient of endogenous respiration or specific maintenance rate	0.13
Monods Constant or Substrate Saturation Constant	1.43
Maximum Specific Growth rate or half the maximum velocity concentration	54.96 d^{-1}
Dilution rate or space velocity	0.2 hr^{-1}
Space time	5.0 hrs

The concentration of Industrial Wastes, Concentration of Biomass, Yield Coefficient, Monods Constant and the maximum specific growth rate were obtained as in Sampson (2021).

A yield coefficient of 0.00016 shows that 0.0016 gram of product was formed per 1 gram of industrial wastes digested by the micro-organisms.

A coefficient of endogenous respiration or specific maintenance rate of 0.13 shows that 0.13 gram of cell was lost due to oxidation of internal storage products or consumption of cellular material to produce cell maintenance energy. A maximum specific growth rate or half maximal velocity concentration of 54.96 per day shows that the maximum concentration of the limiting substrate (Industrial wastes) for growth is averagely 54.96 mg/L for each day of microbial digestion of the industrial wastes.

A Monods Constant or Substrate Saturation Constant of 1.43 shows that 1.43 mg/L of growth rate limiting nutrient is required to achieve half the maximum specific growth rate. A dilution rate or space velocity of 0.2 hr^{-1} shows that 0.2 reactor volume of feed is being fed into the bio-reactor per hour.

A space time of 5.0 hours shows that one bio-reactor volume of industrial wastes is being treated in the bio-reactor every 5 hours.

4.0. CONCLUSION

The volume of a fed-batch bio-reactor is a function of time. This also depends on the volumetric feed rate into the bio-reactor. Kinetic Models for finding the volume of a Fed-batch bio-reactor have been derived. The solution of these kinetic models gave a bio-reactor industrial wastes volume of 25.0m^3 and volume of bio-reactor cylindrical space of 32.35 m^3 for a hydraulic retention time of 5 days.

This shows that 25.0m^3 of industrial wastes was treated every 5 days in a Fed-batch bio-reactor of volume 32.35 m^3.

A Root Mean Square Error (RMSE) of 0.85 shows that 85 % of the variability in the data set was explained by the kinetic models.

5.0. RECOMMENDATION

It is recommended that Fed-batch mode of operation is suitable for the treatment of medium level capacity e.g 5 m^3 per day of industrial wastes as it gives a reasonable bio-reactor industrial waste volume of 25.0 m^3 and a volume of bio-reactor cylindrical space of 32.35 m^3 for a hydraulic retention time of 5 days.

A fed-batch mode of operation is recommended for biological treatment of industrial wastes as change in volume of the bio-reactor is a function of time which also depends on the volumetric feed rate into the bio-reactor.

6.0. REFERENCES

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APPENDIX A

A MATLAB Program in Fourth order Rungekutta for finding the volume of the Bio-Reactor

```
Anaero 14 . m Main Program
Function dN = Anaerovol14 (t,N)
dN = Zeros (3,1);
V = 5.0;
Y = 0.00016;
C1,0 = 0.000015
µm = 54.96;
km = 1.43;
kd = 0.13;
C2,0 = 1.0;
Ve = 1.315159581;
A = 3.80182;
rfi = 1.468;
dN (1) = (v/N(3))* [C1,0 – N(1) + (µm*N(1)* N(2) (Km + N (2)) – kd* N(1)
dN (2) = (v/N(3))* [C2,0 – N(2) + (µm*N(1)* N(2)Y* (Km + N (2)))
dN (3) = v
dN (4) = (ve* A) + rfi
AnaerVol 3.m                                  Execution Program
clear all
clc
format long
t = 0 : 1 : 5
[t Y] = ode45(@Anaerovol14,[t],[0.000015 1.0 0.0015], ‘retol=0.001)
N1 = Y (:,1)
N2 = Y (:,2)
N3 = Y (:,3)
N4 = Y (:,4)
%[t Y 1] = ode23(@stab19,[t],[4 10],
‘reltol = 0.001’)
%N11 = Y 1 (: 1)
%N22 = Y1 (:,2)
ANS[t N1 N2 N3 N4]
```

APPENDIX B

CALCULATIONS

i. Volume of Industrial Wastes to be filled into the Bio-reactor

At t = 1

$V_R(1) = V_e \times 1 \times A + C$

$V_R(1) = 1.315159581 \times 1 \times 3.80182 + 0.0015$

$V_R(1) = 5.001499998 \text{ m}^3$

$V_R(1) \approx 5.00 \text{ m}^3$

At t = 2

$V_R(2) = V_e \times 2 \times A + C$

$V_R(2) = 1.315159581 \times 2 \times 3.80182 + 0.0015$

$V_R(2) = 10.0015 \text{ m}^3$

$V_R(1) \approx 10.00 \text{ m}^3$

At t = 3

$V_R(3) = V_e \times 3 \times A + C$

$V_R(3) = 1.315159581 \times 3 \times 3.80182 + 0.0015$

$V_R(3) = 15.00149999 \text{ m}^3$

$V_R(3) \approx 15.00 \text{ m}^3$

At t = 4

$V_R(4) = V_e \times 4 \times A + C$

$V_R(4) = 1.315159581 \times 4 \times 3.80182 + 0.0015$

$V_R(4) = 20.00149999 \text{ m}^3$

$V_R(4) \approx 20.00 \text{ m}^3$

At t = 5

$V_R(5) = V_e \times 5 \times A + C$

$V_R(5) = 1.315159581 \times 5 \times 3.80182 + 0.0015$

$V_R(5) = 25.00149999 \text{ m}^3$

$V_R(5) \approx 25.00 \text{ m}^3$

From laboratory experimental results, Nigerian FEPA discharge limits were reached on day 5, hence the volume of industrial wastes to be filled into the reactor is taken as the volume on day 5 = 25.0 m^3

ii. Volume of Fed-batch Bio-reactor Cylindrical Space

$$V_r \; cylindrical \; space \; (t) = \; 6.467999998t + 0.00570273$$

At t = 0

$V_r \; cylindrical \; space \; (0) = \; 0.0015 \; m^3$ (Lab level)

$At \;\; t = 1$ (Plant level)

$$V_r \; cylindrical \; space \; (1) = 6.467999998 \; (1) + \; 0.00570273 = 6.473702728 \; m^3$$

$$\approx 6.47 \; m^3$$

$$At \;\; t = 2$$

$$V_r \; cylindrical \; space \; (2) = 6.467999998 \; (2) + \; 0.00570273$$

$$= 12.936 + 0.00570273$$

$$= 12.94170273$$

$$\approx 12.94 \; m^3$$

$$At \;\; t = 3$$

$$V_r \; cylindrical \; space \; (3) = 6.467999998 \; (3) + \; 0.00570273$$

$$= 19.40399999 + 0.00570273$$

$$= 19.40970272 \; m^3$$

$$\approx 19.41 \; m^3$$

$$At \;\; t = 4$$

$$V_r \; cylindrical \; space \; (4) = 6.467999998 \; (4) + \; 0.00570273$$

$$= 25.87770272$$

$$\approx \; 25.88 \; m^3$$

$$At \;\; t = 5$$

$$V_r \; cylindrical \; space \; (5) = 6.467999998 \; (5) + \; 0.00570273$$

$$= 32.34570272$$

$$\approx 32.35 \; m^3$$

iii. Derivation of a constant Dimensionless Value representing the Rate of Reaction

$$rf_i = \frac{\mu_m \, C_1 C_2}{Y(K_m + \; C_2)}$$

Let t represent day

$$rf_i = \frac{54.96 \, t^{-1} \; \times 0.000013 \; \frac{mg}{L} \; \times 0.7 \frac{mg}{L}}{0.00016 \; (1.43 + \; 0.7 \; \frac{mg}{L})}$$

$$rf_i = \frac{54.96 \; \times 0.000013 \; \frac{mg}{L} \; \times 0.7 \; \frac{\cancel{mg}}{\cancel{L}}}{t \; \times 0.00016 \; (1.43 + \; 0.7 \; \frac{\cancel{mg}}{\cancel{L}})}$$

Multiplying both sides by t

$$rf_i(t) = \frac{54.96 \times 0.000013\ \frac{mg}{L} \times 0.7\ \cancel{t}}{\cancel{t} \times 0.00016\ (2.13)}$$

$$rf_i(t) = \frac{5.00136 \times 10^{-4}\,\frac{mg}{L}}{3.408 \times 10^{-4}}$$

$$rf_i(t) = 1.468\ \frac{mg}{L}$$

1000 kg = 1 m^3 approximately

1,000,000 g = 1 m^3

1,000,000,000 mg = 1 m^3

1 m^3 = 1000 Litres

Hence,

1,000,000,000 mg = 1000 L

1000 mL = 1L

Hence,

1,000,000,000 mg = 1,000,000 mL

$$1\text{ mg} = \frac{1{,}000{,}\cancel{000}}{1{,}000{,}\cancel{000{,}000}}$$

$$1\text{ mg} = \frac{1}{1{,}000}\text{ mL}$$

1 mg = 0.001 mL

$$1\,\frac{mg}{L} = \frac{1}{1{,}000}\text{ mL}$$

$$1\,mg = 0.001\text{ mL}$$

$$1\frac{mg}{L} = 0.001\,\frac{mL}{L}$$

$$1.468\frac{mg}{L} = 1.468 \times 0.001\,\frac{mL}{L}$$

$$=0.001468\ \frac{mL}{L}$$

$$rf_i(t) = 0.001468\ \frac{mL}{L}$$

The Volume Correction Factor measured at 75 ^{o}C = 2×10^{-6}

$$0.001468\ \frac{mL}{L} \times \frac{1}{2 \times 10^{-6}}$$

$$0.001468\ \frac{mL}{L} \times\ 500{,}000$$

$$= 734\ \frac{mL}{L}$$

$$1\ mL = 0.001\ L$$

$$1\ \frac{mL}{L} = 0.001\ \frac{L}{L}$$

$$734\ \times 0.001 = 0.734\ \frac{L}{L}$$

= 0.734, a dimensionless number

Considering safety concerns, e.g. pressure build-up in the bio-reactor, multiply this by a factor of safety of 2

$0.734\ \times 2 = 1.468$, a dimensionless number.

$$rf_i(t) =\ 1.468$$

t only indicate that rf_i is a function of time.

iv. Root Mean Square Error (RMSE) for the Concentration of Industrial Wastes

$$\frac{\sum(C_2,Exp - C_2,model\ )^2}{n}$$
$$= \frac{3.631690309}{5}$$
$$= 0.7263380618$$
$$\cong 0.73$$
$$\sqrt{0.73} = 0.85$$

v. Root Mean Square Error (RMSE) for Concentration of Biomass

$$\sum(C_1, Exp - C_1, model)^2 =\ 2.432283462$$
$$\frac{\sum(C_1,Exp - C_1,model\ )^2}{n}$$
$$= \frac{2.432283462}{5}$$
$$= 0.4864566924$$
$$\sqrt{0.4864566924} = 0.6974644739$$
$$\cong 0.70$$

vi. Cross Sectional area and Velocity of the Industrial Wastes

V_e, the velocity of the Industrial Wastes

$$= \frac{Volumetric\ feedrate}{Area}$$

Volumetric federate = 5 m^3/d

Cross sectional area of the Industrial wastes $=\frac{\pi D^2}{4} = \frac{3.142 \times (2.20)^2}{4}$

$$= \frac{3.142 \times 4.84}{4} = \frac{15.20728}{4}$$

$$= 3.80182 \text{ m}^2$$

$$\text{Hence, velocity} = \frac{5}{3.80182}$$

$$= 1.315159581 \text{ m/d}$$

vii. Height of Fed-Batch Bio-Reactor

Height of Fed-Batch Bio-Reactor

$$= \frac{4V_R\ Cylindrical\ Space}{\pi\ D^2}$$

$$= \frac{4 \times 32.35}{3.142 \times (2.20)^2} = \frac{129.40}{15.20728}$$

$$= 8.509082492\ m$$

$$\approx 8.50\ m$$

viii. Height of Fed-Batch Bio-reactor Industrial Wastes

Height of Fed-Batch Bio-Reactor Industrial Wastes

$$= \frac{4V_R}{\pi\ D^2} = \frac{4\ \times 25}{3.142\ \times (2.20)^2}$$

$$= \frac{100}{15.20728}$$

$$= 6.575797907\ m$$

$$\approx 6.60\ m$$

ix Dilution Rate or Space Velocity

$$\frac{V}{V_R} = \frac{V_e \times A}{V_R}$$

$$= \frac{1.3\ \times 3.8}{25.001500000000014}$$

$$= \quad 0.1975 \ \underline{\sim} \ 0.2$$

x. Inverse of Dilution Rate or Space Time

$$\text{Space Time} = \frac{1}{Space\ Velocity}$$

$$= \frac{1}{0.2} = \text{5.00 hours}$$

APPENDIX C

NOMENCLATURE

Symbol		Unit
A	Cross Sectional Area of Industrial Wastes	m^2
$C_{1,}$	ConentrationBiomas	mg/L
$C_{1,0}$	Initial Concentration of Bio-mass	mg/L
C_2	Conentration Industrial Wastes	mg/L
$C_{2,0}$	Initial Concentration of the Industrial wastes	mg/L
k_d	The Coefficient of Endogenous respiration or the Specific Maintenance Rate	(d^{-1})
K_m	Monods Constant or Substrate Saturation Constant	$kmol/m^3$
h	Height of reactor Industrial Wastes	m
ρ	Density of Industrial wastes	mg/L
rf_i	Rate of Reaction of Component 'i'	$mgl^{-1}s^{-1}$
t	Time	days
μ_m	Maximum Specific Growth Rate or Half Maximal Velocity Concentration	d^{-1}
V	Volumetric Feed rate	m^3d^{-1}
V_e	Velocity of the Industrial Wastes	md^{-1}
V_r	Volume of Bio-reactor Cylindrical Space	m^3
V_R	Volume of Bio-reactor Industrial Wastes	m^3
Y	Yield Coefficient	-

ABBREVIATIONS

Abbreviation	**Meaning**
FBB	Fed-Batch Bio-reactor
FEPA	Federal Environmental Protection Agency
MATLAB	Matrix Laboratory
VCF	Volume correction Factor