IJAAR PUBLISHING®
DOI: 10.46654

International Journal of Advanced Academic Research | ISSN: **2488-9849**
Vol. 7, Issue 10 (October, 2021) | www.ijaar.org

TIME-VARYING VOLATILITY MODELING OF STOCK RETURNS DURING COVID-19: THE NIGERIA EMPIRICAL EVIDENCE

Agbam, Azubuike Samuel
Department of Banking and Finance
Faculty of Management Sciences
Rivers State University,
Port Harcourt, NIGERIA.
Phone: 0706 738 1059
Email: azubuikesamuelagbam@yahoo.com

Abstract

This study models time-varying volatility in the Nigerian Stock Exchange (NSE), investigating whether it has been affected during the COVID-19 periods. We examined the persistence of volatility and the presence of leverage effects in Nigerian equity market before and during the periods of COVID-19. It was found that there is GARCH effects in the stock market before and during the COVID-19 periods. However, volatility was pooling and spiky more during the COVID-19 giving the verdict that the market has become highly unpredictable during this period. In addition, leverage-effects are more pronounced during the pandemic peril.

Keyword: Volatility; conditional volatility, GARCH; COVID-19 crisis

1. Introduction

The degree of fluctuation in asset prices is referred to as the variability or randomness of asset prices. Furthermore, variability is also described as the rate and magnitude of price changes (which in finance is referred to as a risk). In recent years, change in volatility of stock returns has been of great concern because it is being used for measuring financial risks.

The variability of asset prices which is understood as volatility of financial market returns is induced by changing investors' expectations due to flow of information to the market. Such information (positive or negative) may be macroeconomic issues such as changes in inflation rates, growth rates, government policies and interest rates, sector or company specific such as foreign investments, mutual funds, investment trends, periodical reports, mergers and acquisitions, dividend declarations etc.

The standard deviation of returns is regarded as volatility, and it measures return distribution from the mean. Volatility is a measure of the intensity of random or unpredictable changes in asset returns and it is a fundamental characteristic of financial markets whose measuring and forecasting has always been important, but even more so in the current crisis.

High volatility means that there is wide range fluctuation in prices over short term period, while low movement of prices indicates low volatility. Time-varying volatility refers to the fluctuations in volatility over different time periods. Investors may choose to study or consider volatility of an underlying security during various time periods. For instance, the volatility of certain assets may be lower during the summer when traders are on vacation. Time-varying volatility describes how the price volatility of an asset may change given different time periods.

Conditional volatility is used to describe the time-varying model. A conditional distribution is therefore a distribution that governs a return at a particular instance in price. Alexander (2001) stated in a more general term that a conditional distribution is any distribution that is conditional on a set of known values for some of the variables, that is, on information set.

Modeling and forecasting stock market volatility is of considerable interest to the practitioners and researchers alike. This has led to considerable research in this area in the recent past. Starting with the pioneering work of Mandelbrot (1963) and Fama (1965), various features of stock returns have been extensively documented in the literature which are important in modeling stock market volatility. It has been found that stock market volatility changes with time (i.e., it is 'time varying') and also exhibits positive serial correlation or 'volatility clustering' (Mandelbrot, 1963). This implies that the changes in volatility are non-random. Moreover, the volatility of returns can be characterized as a long-memory process as it tends to persist (Bollerslev, Chou and Kroner, 1992). Volatility shocks at daily or lower frequency tend to decay very slowly (Andersen and Bollerslev, 1996). It was Black (1976) who first noted that even the changes in volatility have been found to be negatively correlated with changes in the stock prices. This has been termed as 'leverage effect in volatility.' He argued that the changes in stock volatility are too large in response to changes in return direction to be explained by the leverage effect alone.

The ARCH/GARCH specifications have been used to model the time-varying volatility of stock returns since after the highly influential papers of Engle (1982) and Bollerslev (1986).

While most of these studies focused matured markets, a few have provided evidences for emerging markets. Prominent among them are: Lee et al. (2001) examined volatility in the Chinese stock market, and Olowe (2009) and Kaur (2004) in the Nigerian and Indian ones respectively. Other authors (Bekaert and Harvey, 1997; Aggarwal et al. 1999; Mookerjee and Yu, 1999; Lee et al. 2001; Kaur, 2004; etc.) have examined time varying-volatility models using daily data for other emerging countries. Wei (2002) examined Chinese weekly data instead. Zhou and Zhou, (2005) tested for co-integration between Chinese stock markets using daily data before and after Hong Kong's return to China. Finally, Tripathy and Gil-Alana (2010) carried out a volatility forecasting exercise for the Indian stock market using five different models: (i) Historical/Rolling Window Moving Average Estimator, (ii) Exponentially Weighted Moving Average (EWMA), (iii) GARCH models, (iv) Extreme Value Indicators (EVI) and (v) Volatility Index (VIX). Their results suggest that the EVI model (followed by the GARCH and VIX models) have the best forecasting properties.

A GARCH model introduces more detailed assumptions about the conditional distributions of returns instead of modeling the data after they have been collapsed into a single unconditional distribution. With time-varying volatility, returns are assumed to be generated by a stochastic process in GARCH specification. These conditional distributions change over time in an auto-correlated way, and the conditional variance, is in itself an autoregressive process.

The GARCH models are based on a statistical theory that is justified by empirical evidence. There is no need to impose unrealistic assumptions to force it into a framework that is inconsistent with its basic assumptions. This consistency has led to many GARCH models to measuring financial risks and pricing, thereby making the model have an edge over other models.

This study directed on the Nigerian Stock Exchange (NSE), one of the largest stock exchanges in Africa by market capitalization and the daily turnover, and number of trades for equities. Furthermore, while trying to investigate whether the Nigeria stock market is affected by the recent COVID-19 pandemic, this study provides additional evidence on the volatility behaviour of the stock market. The specific objectives are: (i) to estimate the degree of volatility in the closing values of The Nigerian Stock Exchange, (ii) to examine the leverage effect in the Nigerian stock market before and after the Pandemic; and (iii) to forecast the future volatility in the Nigerian Stock Exchange return series. The hypotheses to be tested are: (1) they do not exhibit asymmetric volatility, (2) their volatility has been higher during the crisis period and (3) there is no leverage effect in the Nigeria stock market. The remainder of the paper is organized as follows: section 2 is the literature review, and section 3 describes the data and methodology. Section 4 shows the analysis and results while Section 5 offers some concluding remarks.

2. Literature Review

Stock markets prices are highly fluctuating, (exhibit volatility); thereby bringing variation in the returns of the investors. The changing price is as a result of active participation of investors. The more the investors buy a stock than they sell a stock, the prices will go up, while the more they sell than they buy – the prices will go down. The movement of the prices determines the return and volatility of the market. It is more important to know the factors that influence them, what causes the investors to conduct themselves way they do.

The collection of small and large movements of asset prices is referred to as volatility clustering, and it features prominently in asset prices volatility. While examining the characteristics of return volatility, Boss (2007) observed that large changes tend to be followed by large changes of either sign, and small changes tend to be followed by small changes. This volatility clustering is believed to be caused by behavioural switching. And each asset price volatility and historical returns eventually will revert to the long-run mean or average level of the entire database (mean reversion). This effect of shocks for an underlying asset will take a long time to recover to its normal mean level, thus such return series is characterized by a level of volatility persistence.

The world is said to be a global village, and the markets are highly integrated; therefore, the volatility of one asset market consistently spreads over to other asset market and any market disturbances in developed countries are transmitted, causing higher volatility across other global markets.

In his examination of volatility trend of Indian stock market from 1996 to 2005 using daily and monthly data of returns Rakesh Kumar (2007) observed high volatility during the decline period (1996-1999) and recession period (2000-2002) and moderately less volatile during the economic growth period (2003-2005), since the investors are largely response to economic aspects.

Variability or randomness of asset prices is associated with the flow of information; and it is a vital factor in the movements of the stock prices (Ross, 1989). Ramanthan and Gopalakrishan (2013) suggested several other factors in addition to latest information that influence the price movements such as inflation, economic strength of market and peers, psychological issues, supply and demand (liquidity) and uncertainty of company's future (even without information).

The effect of information flow on the behaviour of stock prices was investigated by Jones, Kaul, Lipson (1994). It was suggested that the public information is the major source of short-term return volatility. They further stated that the investors' interpretation and reaction to the immediate information and the adjustment of the market prices up or down leads to high fluctuations (volatility) in the market. Furthermore, it was also suggested that trading volume and trading opportunities and various market and non-market components are associated with volatility.

The pattern of volatility persistence is one most significant characteristic of volatility (Berry and Howe, 1994). They found that the relationship between public information and trading volume is positively reasonable but the relationship between price and volatility is insignificant.

They analyzed a one-year time series of five-minute Deuchemark-U.S. Dollar exchange rates was undertaken by Andersen and Bollerslev (1997), it was discovered that the degree of intra-daily returns volatility persistence was high and the frequency of inter-daily returns was low. But Foster and Viswanathan (1993) discovered that during the first half hour of the day, using intraday data of New York Stock Exchange, the actively traded firms, trading volume, adverse selection costs and return volatility were high.

One would be tempted to say and rightly too that since the behaviour of stock returns volatility is influenced by economical and institutional changes, the market liberalization also have an impact on the patterns of volatilities of stock returns. The world being a global village, the stock markets are integrated with each other and thus the information flows are directly associated with the variance of price changes. With the increase of rate of flow of information across the markets, the market liberalization can increase or decrease the volatility in the market. And when markets are open more information is released than when they are closed and the stock prices are more volatile in the period when the markets are open for trading (French and Roll, 1986), and the stock prices are more volatile in the period when the markets are open for trading. But in a study conducted by Sa Young Lee (1998) it was discovered that market liberalization does not impact on volatilities of stock returns in the Korean market and the conditional heteroskedasticity of variance in the Korean stock market index return series was studied for the period between 1977 to 1994.

A class of models of volatility is required in measuring the associated risk. The ARCH (Autoregressive Conditional Heteroscedasticity) and GARCH (Generalized Auto regressive Conditional Heteroscedasticity) are the broadly accepted models of volatility (Bose, 2007). The models estimates the variance of the forecasted return based on past forecast errors as well as past estimates of volatility.

Volatility clustering and leptokurtosis (fat tails) are commonly observed in financial time series. More also observed in the financial returns is the leverage effects which occurs when the change in stock prices are negatively correlated with the changes in volatility. As a result of these, several financial time series analysts use varying variance models to estimate and predict volatility of stock prices. Engle, (1982) proposed time-series conditional variance using lagged disturbance with Autoregressive Conditional Heteroskedasticity (ARCH). The result reveals that in order to capture the dynamic behaviour of conditional variance, a high order ARCH model is required. In solving this problem, in 1986 a Generalized ARCH model (GARCH) was developed which reduces the number of estimated parameters from infinity to two based on infinite ARCH specifications called a Generalized ARCH model (GARCH)

It was argued by Hansen and Lunde (2005) that GARCH (1, 1) works quite well in estimating volatility of financial returns as compared to more complicated models. Hung-Chung et al.(2009) modeled volatility of Chinese stock market, and it was revealed that GARCH model with an underlying leptokurtic asymmetric distribution has better forecasting ability as compared to an underlying normal distribution; GARCH (1, 1) model with a fat tail error distribution leads to an improvement in a Volatility forecast (Wilhemsson, 2006), and Student's t distribution as a distribution assumption to a GARCH model outperforms the exponential distribution and a mixture of Normal distribution (Chuanga et al. (2007).

In another development, Balaban (2004) compared the forecasting ability of symmetric and asymmetric GARCH models [(GARCH (1, 1), GJR-GARCH (1, 1) and EGARCH (1, 1)] using volatility equations and discovered that EGARCH model performs better on the out of sample data forecast, followed by GARCH and GJR-GARCH (1, 1) performed poorly.

Furthermore, from the observations from Egypt (CMA and general index) and Israel (TASE-100 Index), Floros, (2008) modeled volatility, and forecasted market risks using GARCH. The Egyptian CMA Index was found to be the most volatile series due to prices (and economy) uncertainty during the time period under consideration. Rafique and Kashif-ur-Rehman, (2011) also studied volatility clustering, excess kurtosis and heavy tails of time series of KSE using ARCH and GARCH. The results show that ARCH and GARCH model capture volatility clustering and lepkurtosis but they fail to model leverage effects because their distribution is symmetric. Again, GARCH do not always fully embrace the thick tails property of a high frequency financial time series data. Non-linear extension of GARCH model such as the exponential GARCH (EGARCH) and Asymmetric Power ARCH (APARCH) was proposed by Nelson (1991) and Ding et al, (1993) respectively, to address these problems.

It should be noted that investors respond more sensitively to bad news rather than good news, which cause high volatility in the stock prices, and studies show that there are asymmetrical changes in the stock prices for a given event or shock leads to considerably higher volatility in the stock returns.

The leverage effect refers to the observed tendency of an asset's volatility to be negatively correlated with the asset's return – typically, rising asset prices are accompanied by declining volatility and vice versa. In his investigation on leverage effect on volatility, Black reports that there is a strong negative correlation between stock price change and volatility return - stocks volatility likely to be increase when stock prices declines. The leverage effect suggests that if there is a decrease in stock price of a company, it reduces the value of equity comparative to debt, and increases the financial leverage and thus increases the risk of holding the equities, which in turn increase the future volatility. But asymmetric volatility response cannot be influenced by leverage alone (Schwert (1989).

The movement of information in the market and the volatility of one stock market spill-over to another stock market are transmitted all around the world where there are strong financial market linkages. Kumar and Mukhopadyay (2002) is among the several empirical studies that have proved and suggested that, 'volatility shocks in the developed stock markets have significant impact on the returns and volatility spillover effects on emerging stock markets including India' as they reported that there is a significant relationship in return and volatility spillover from US to India by using Granger causality test and Univariate GARCH models.

Kim (2005), Wang (2005), Mukherjee and Nath (2008), Yu and Hassan (2008), etc. also examined the volatility spillover effect between the developed markets with US markets, and reported a unidirectional volatility spillover from US markets to other markets. More also, while examining the mean and volatility spillover effect from Brazil, Russia, India and China (BRIC) countries stock market to world markets and to regional stock indices, it was reported that the world stock index have an effect on the mean returns and volatility of BRIC countries stock indices and regional markets have more volatility spillover effect compared to world markets (Bhar and Nikolova (2007).

The investigation of market integration and volatility spillover effect between Indian stock market and 12 other Asian stock markets was carried out by Mukherjee and Mishra (2008), and reported bi-directional intra-day spillover effect between Indian stock market and its counterparts and there was no immediate communication of information between the markets, as a bi-directional spillover effect between the Indian stock market and US (foreign exchange market).was also found by Mishra et al. (2007). The daily data of S&P CNX Nifty index from June 2000 to March 2007 was used by Suchismita Bose (2007), an increasing volatility during the times of market decline (2000-2002) being sharper and more persistent was found, with a significant asymmetry response. On the same vein, Singh et al. (2008) observed volatility spillover effect from Hong Kong, Japan, Korea, Singapore and US to the Indian market as they investigate the price and volatility spillover between India and other markets.

The daily closing values of S&P 500 (US market) and S&P CNX Nifty (Indian) indices from January 1996 to September 2008 was studied, and a significant asymmetry response on returns and spillover effects on volatility was reported by Badhani (2009) The returns in Indian stock market are more responsive to negative shocks in US market than the positive shocks and the positive shocks effect on the volatility.

The investigation of the relationship between Sensex, Dow Jones, FTSE, BVSP, MerVal and JKSE carried out by Sarkar, Chakrabarti and Sen (2009) revealed that the volatility in the developed market indices affects the Sensex volatility; proved a worldwide contagion – shocks, which initially affect only a few particular region/country spread to the rest of the regions/countries. It was also reported that Sensex volatility is also correlated to some degree of the volatility of the Jakarta Stock index, indicating toward some kind of regional contagion and impulse-response function shows, a shock in Dow Jones, the Jakarta stock index and BVSP have a reflective effect on Sensex.

The spillover effect between US and Major European stock markets was studied by Al-Zeaud and Al-shbiel (2012). Reports reveal that there is unidirectional volatility spillover effect from Frankfurt to Paris, and Paris to London; where bad news induced volatility are transmitted more strongly the volatility declines - spillover effect from London market to New York, Paris and Frankfurt stock markets, and within European stock markets.

The dynamic return and volatility spillover as well as bi-directional contemporaneous from US Stock Market to Indian stock markets and vice versa was examined by Sen and Bandhopadhyay (2012) and Japan and UK to India by Sakthivel et.al (2012). It was reported that there was bidirectional volatility spillover between S&P 500 and BSE Sensex, and uni-directional volatility spillover from Japan and UK to India (Sakthivel et.al (2012).

The volatility spillover effect during pre-crisis, post-crisis, in-crisis periods between rupee-dollar exchange rate and CNX return series was examined by Panda and Deo (2014). It was reported that there was asymmetry and volatility spillover in all the three periods. Compared to two other periods, the asymmetry was higher, as well as volatility spillover effect during the post-crisis period.

3. Data and Methodology

3.1 Data

We employed raw daily data of All-share index for Nigerian equity market. The data are obtained from www.investing.com to cover a period of 5th January 2015 to 28th September 2020, about 1408 data points. We further break the period to pre C-19 period and C-19 period in Nigeria. The pre-C-19 ranges from 5th January 2015 to 2ndFebruary 2020, while C-19 period takes effect from 2nd March 2020 to 28th September 2020 in Nigeria. The raw price data are log differenced to show the pooling of volatility in the pre-C-19 and C-19 periods. This is shown in the figures below.

3.2 Methodology

The fundamental idea in GARCH is to add a second equation to the standard regression model: the conditional variance equation (Enders 2004). This equation describes the evolution of the conditional variance of the unexpected return process, $V_t(\varepsilon_t) = \sigma_t^2$.The dependent variable, the input to the GARCH volatility model, is always a return series, and accordingly a GARCH model consists of two equations. The first equation is the conditional mean equation. This can be anything, but since the focus of GARCH is on the conditional variance equation it is usual to have a very simple conditional mean equation. Many of the GARCH models used in practice take the simplest possible conditional mean equation $r_t = c + \varepsilon_t$, where c is a constant. In this case the unexpected return ε_t is just the mean deviation return, because the constant will be the average of returns over the data period. In some circumstances it is better to use a time-varying conditional mean, but on the other hand, using to many parameters in the conditional mean equation might lead to convergence problems.

If there is significant autocorrelation in returns, autoregressive moving average conditional mean should be used to model the returns. The second equation in a GARCH model is the conditional variance equation. Different GARCH models arise because the conditional variance equations are specified in different forms. There is a fundamental distinction between the symmetric GARCH models that are used to model ordinary volatility clustering and the asymmetric GARCH models that are designed to capture leverage effects. In symmetric GARCH the conditional mean and conditional variance equations can be estimated separately. This kind of estimation is not possible for asymmetric GARCH models making their estimation more complex (Alexander 2001). Underlying every GARCH model there is also an unconditional returns distribution. The unconditional distribution of a GARCH process will be stationary under certain conditions imposed on the GARCH parameters and if necessary these conditions can be imposed on the estimation.

4. Empirical Analysis and Results

Figure 1 Pre-C-19 Period

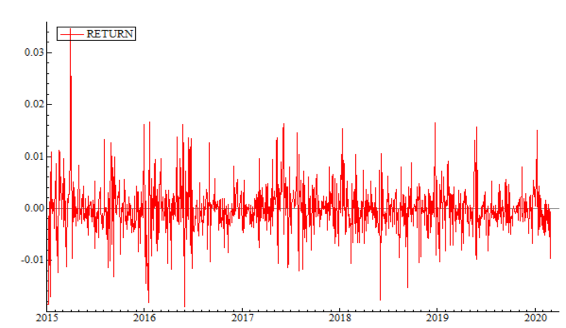
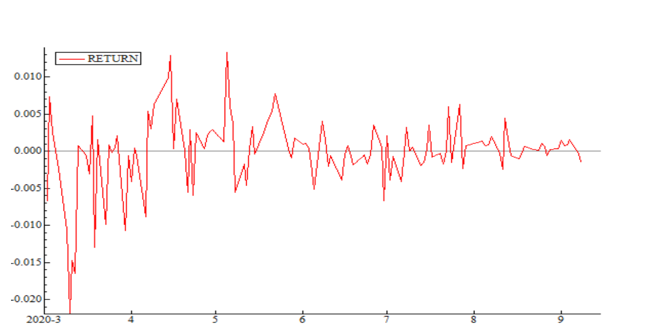


Figure 2 C-19 Period



We have evidence of more spiky volatility in return during C-19 period than in the pre C-19 period. Equally, observed that the market exhibits volatility pooling, heteroscedasticity and constant mean in both periods. Other characteristics of the data are shown in table 1.

Table1
Descriptive Statistics for NSE Return

Statistics Period	Pre-C-19 Period	C-19
Mean	-8.78E-05	-4.22E-05
Median	-0.000257	0.000292
Std. Dev.	0.004570	0.004797
Skewness	0.430902	-1.193513
Kurtosis	8.120876	7.671846
Jarque-Bera	1434.820*	146.7949*
Sum Sq. Dev.	0.026653	0.002923
Observations	1277	128

Source: Computed by the Authors
**1% Level of Significance*

The mean of return is negative for the periods of C-19 and pre C-19. However, the C-19 period has higher volatility than the pre C-19. Before the advent of C-19 in Nigeria market return is positively skewed, while during the C-19 period, market return become negatively skewed. In both periods, the market return is leptokurtic, highly picked and concentrated around its mean value. Likewise, in these periods, return does not follow a normal distribution process.

Table 2
Estimates of the ARCH-LM Test for the NSE Return

Statistics	Pre C-19	C-19
F-statistic	31.46684*	4.49636**
Chi-squared	30.75493*	4.407626**

Source: Computed by the Authors
**1% Level of Significance **5% Level of Significance*

At 1% and 5% respectively for the pre C-19 and C-19 periods, the assumption of homoscedasticity is rejected. Supporting the presence of heteroscedasticity revealed in figures 1 and 2 respectively.

Table 3
Estimates of the Unit Root Test for the NSE Return

ADF-tset	KPSS-test			
	Pre C-19	C-19	Pre C-19	C-19
Test-value	-26.33*	0.13*	-4.7*	0.23*
1% level	-3.435263	0.739000	-3.48288	0.739000
5% level	-2.863597	0.463000	-2.88448	0.463000
10% level	-2.567915	0.347000	-2.57908	0.347000

Source: Computed by the Authors
**Significant at all levels*

The unit root test results yielded by ADF and KPSS show a massive rejection of the null hypothesis at all level of significance for both periods. This means that the market return does not exhibit unit root before the C-19 and during the C-19. Additional evidence of fractional integration is provided in table 4.

Table 4
Test of Fractional Integration for the NSE Squared Return

	Pre C-19	C-19
d-figarch	0.18**	0.83**

Source: Computed by the Authors
***5% Level of Significance*

The test reveals that the market is non-random in any of the period. However, the market return is more fractionally integrated during the C-19 period than in the period before the pandemic. This provides the basis for examining the Nigerian equity market for volatility pooling and asymmetric tendency using the family of GARCH models.

4.2 The GARCH Model

We follow the spirit of Bollerslev (1986) to introduce GARCH (1 1) model for this study. The formal definition of this model is.

$$\delta_t^2 = a_1 + a_2\delta_{t-1}^2 + a_3 e_{t-1}^2 , \quad (1)$$

Where: δ_t^2 conditional variance proxy by squared return, δ_{t-1}^2 GARCH term, e_{t-1}^2 ARCH term, a_1 non-negativity constant, must be greater than zero, the slope parameters (a_2 and a_3) are non-negativity, they can equal zero. The estimated results of the GARCH(1 1) model is shown in table 5.

Table 5
Estimated GARCH Results for NSE before and during C-19

Regressors	Pre-C-19	C-19
ARCH	0.42*	0.31*
GARCH	0.34*	0.81*
ARCH + GARCH	0.76	1.12

Source: Computed by the Authors
**1% Level of Significance*

The results show that there is presence of ARCH effects in the market for the two periods, but the effects were stronger before the C-19, meaning that volatility reacts more to the movements in the market before C-19 than during the C-19. Conversely, the GARCH effects were stronger during the C-19 period, suggesting that shocks are more likely to occur in the future during the C-19. The sum of the coefficients of the two terms is larger during the C-19 than the pre C-19, meaning that volatility persists longer time during the C-19 period. Another important question addressed in this study is whether bad news results in higher volatility than good news. To answer this question we estimate the EGARCH (1 1) and TGARCH (1 1) models

4.3The EGARCH Model

Nelson (1991) introduced the EGARCH model to account for asymmetric effects in stock markets. In akin to this author, we define EGARCH (1 1) model as follows.

$$\ln\delta_t^2 = b_1 + b_2\ln\delta_{t-1}^2 + b_3 e_{t-1}^2\delta_{t-i}^{-1} + b_1[|e_{t-1}|\delta_{t-i}^{-1} - \sqrt{2/\rho i}] \quad (2)$$

Where: $e_{t-1}^2\delta_{t-i}^{-1}$ is asymmetric term (that is the standardized lag squared errors) and the coefficient is b_3 which must be less than zero and significant for the presence of asymmetry or leverage effects. The test result on leverage effects is presented in table 6.

Table 6
Estimated EGARCH Results for NSE before and during C-19

Regressors	C-19	Pre-C19
ARCH	0.31*	0.02*
GARCH	0.35*	0.91*
Asymmetry	-2.23*	-0.46

Source: Computed by the Authors
**1% Level of Significance*

In addition to the ARCH and GARCH effects, there is evidence of leverage effects in the market only before the outbreak of C-19 pandemics. During the Pandemic, we observe that there is no leverage effect, opposing the position (bad news results in higher volatility compared to good news) in the market before the C-19. We corroborated this result by estimating the Threshold GARCH(1 1) of Glosten et al. (1993). The format of this model can be expressed as.

$$\delta_t^2 = c_1 + c_2 \ln \delta_{t-1}^2 + b_3 e_{t-1}^2 + b_4 e_{t-1}^2 D_{t-1} \tag{3}$$

Where: D_{t-1} is a dummy variable adopting the value of 1 when $e_{t-1} \leq 0$, 0 when $e_{t-1} > 0$. The asymmetric coefficient is b_4. The model estimated result is reported in table 7

Table 7
Estimated TGARCH Results for NSE before and during C-19

Regressors	C-19	Pre-C-19
ARCH	3.14	0.20
GARCH	-2.35	-0.32
Asymmetry	0.74*	1.01*

Source: Computed by the Authors
**1% Level of Significance*

4.4 The TGARCH Model
According to the result yielded by the TGARCH model, there is presence of leverage effects in the market for both periods. However, the effects are more pronounced during the C-19, which suggests that in this current period, bad news has greater impact on volatility than the prior period. We now estimate the loss function to detect the model with the most forecasting ability. The results are shown in table 8.

Table 8
Estimates of the Loss Functions

Model	Pre C-19		C-19	
	RMSE	MAE	RMSE	MAE
GARCH	0.000059	0.00002	0.00006	0.000023
EGARCH	0.000059	0.00002	0.000063	0.000022
TGARCH	0.000059	0.00002	0.000063	0.000023

Source: Computed by Authors

Before the C-19 period, the competing models performed equally as indicated by the root mean squared errors (RMSE), and mean absolute error (MAE) statistics, while during the pandemics, EGARCH model is better.

5. Conclusive Remarks

In this study, we have examined the persistence of volatility and the presence of leverage effects in Nigerian equity market before and during the periods of C-19. We found out that there is GARCH effects in the market before and during the C-19 periods. However, volatility was pooling and spiky more during the C-19 giving the verdict that the market has become highly unpredictable during this period. In addition, leverage-effects are more pronounced during the pandemic peril.

Reference

Aggarwal, R., Inclan, C., & Leal, R., (1999). Volatility in emerging stock markets. *Journal of Financial and. Quantitative Analysis 34(1).*

Alexander, C. (2001). *Market models: A guide to financial data analysis*. New York: John Wiley & Sons. 514 p. ISBN13: 9780471899754.

Andersen, T. G., and Bollerslev, 1997, "Hetergeneous information arrivals and return volatility dynamics: Uncovering the long-run in high frequency returns", *Journals of Finance 52, 975-1005.*

Badhani, K. N. (2009). Response Asymmetry in Return and Volatility Spillover from the US to Indian Stock Market, *The* IUP *Journal of Applied Finance, Vol. 15, No.9, 2009.*

Balaban, E. (2004). Forecasting exchange rates volatility. Working paper. URL: http://ssrn.com/abstract=494482.

Bekaert, G., & Harvey, C.R., (1997). Emerging equity market volatility. *Journal of Financial Econometrics. 43 (1),* 29–77.

Berry, T. D., & Howe, K. M. (1994). Public information arrival, *Journal of Finance 49, 1331- 1346.*

Bhar, R. & Nikolova, B. (2007). Analysis of Mean and Volatility Spillovers using BRIC Countries Regional and World Equity Index Returns", *Journal of Economic Integration, Vol.22, No.2, PP. 369-381.*

Black, F., (1976). Studies in stock price volatility changes, Proceedings of American Statistical Association, *Business and Economics Statistics Section, 177-181.*

Bollerslev, T., (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics. 31 (3),* 307–327.

Bollerslev, T, Chou, R Y and Kroner, K F (1992). ARCH Modeling in Finance: A Review of Theory and Empirical Evidence, *Journal of Econometrics,* 52(1-2), 5-59.

Bollerslev, T. R., & Wright, J. .H., (2000). Semiparameteric estimation of long memory volatility dependencies: the role of high frequency data. *Journal of. Econometrics. 98,* 81–106.

Chuanga, I. Y., Lub, J. R. & Leea, P. H. (2007). Forecasting volatility in the financial markets: a comparison of alternative distribution assumptions: *Applied financial Econometrics 17:* 1051 – 1060.

Dickey, D., Fuller, W.A., (1979). Distributions of the estimators for autoregressive time series with a unit root. *J. Am. Stat. Assoc. 74,*

Ding, Z., Granger, C.W.J., & Engle, R.F., (1993). A long memory property of stock market returns and a new model. *Journal of Empirical. Finance 1,* 83–106.

Enders, W. (2004). *Applied Econometric Time Series*. 2nd edition, New York, John Wiley & Sons, 480 p. ISBN: 978-0-471-23065-6.

Engle, R.F., (1982). Autoregressive conditional heteroscedasticity with estimates of the variance. *Econometrica 50 (4),* 987–1007.

Fama, E F (1965). The Behavior of Stock-market Prices, *Journal of Business,* 38(1), 34-105.

Floros, C., (2008). Modeling Volatility using GARCH MODELS: Evidence from Egypt and Israel, *Middle Eastern Financial and Econometrics, 2:* 31 – 41.

Foster F. D., & Viswanathan, S. (1993). The effect of public information and competition on trading volume and price volatility", *Review of Financial Studies 6, 23-56.*

French, K. R., & Roll, R. (1986). Stock return variance: The arrival of information and the reaction of traders. *Journal of Financial Economics 17, 5-26.*

Gil-Alana, L.A., (2008). Fractional integration and structural breaks at unknown periods of time. J. Time Ser. Anal. 29, 163–185.

Gil-Alana, L.A., & Tripathy, T., (2012). Long range dependence in the Indian stock market. Evidence of fractional integration, nonlinearities and structural breaks (Preprint).

Glosten, L.R., Jagannathan, R., & Runkle, D.E., (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *Journal of Finance 48 (5),* 1779– 1801.

Hansen, P. R. and Lunde, A. (2005a). A forecast comparison of Volatility Models: Does anything Beat a GARCH (1, 1)? *Journal of Applied Econometrics 20:* 873 – 889.

Hung – Chung, L., Yen – Hsien, L., & Ming – Chih, L. (2009). Forecasting China stock markets volatility via GARCH Models under skewed- GED distribution. *Journal of money, Investment and Banking,* 542 – 547.

Jones, C M., Kaul, G. & M. L Lipson, M. I. (1994). Information, trading, and volatility, *Journal of Financial Economics 36, 127-154.*

Kaur, H., (2004). Time varying volatility in the Indian Stock Market. Vikalpa 29 (4), 25–42.

Lee, C.F., Chen, G.M., & Rui, O.M., (2001). Stock returns and volatility on China's stock markets. *Journal of Financial Research. 24 (4),* 523–544

Kim, S. J. (2005). Information Leadership in the Advanced Asia-Pacific Stock markets: Return, Volatility and Volume Information Spillovers from the US and Japan", *Journal of the Japanese & International Economies, 19 (3), 338-365.*

Mandelbrot, B (1963). "The Variation of Certain Speculative Prices," *Journal of Business,* 36(3), 394-419.

Mookerjee, R., & Yu, Q., (1999). An empirical analysis of the equity markets in China. *Rev. Financial Economics 8 (1),* 41–60.

Mukherjee, K. & Nath Mishra, (2008). Stock market integration and volatility spillover: India and its major Asian counterparts, *Munich Personal RePEc Archive P. No. 12788*

Nelson, D.B., (1991). Conditional autoregressive conditional heteroskedasticity in assets returns: A new approach. *Econometrica, 59:* 347 – 370

Olowe, R.A., (2009). Stock return, volatility and the global financial crisis in an emerging market: the Nigerian case. *Int. Rev. Bus. Res. Pap. 5 (4),* 426–447.

Phillips, P.C., & Perron, P., (1988). Testing for a unit root in time series regression. *Biometrika 75 (2),* 335–346.

Rafique, A. & Kashif – Ur – Rehman (2011). Comparing the persistency of different frequencies of stock returns volatility in an emerging markets: A case study of Pakistan. African journal of Business management 5: 59 – 67.

Ramanathan and Gopalakrishnan, (2013). Volatility in Indian Stock Market – A Study of post and pre recession period, *Namex International Journal of Management Research 134 3 (1), Jan.*

Ross S A, (1989). Information and volatility: The no-arbitrage martingale approach to timing and resolution irrelevancy, *Journal of Finance 44, 1-17*

Sakthivel, P., & Bodkhe, N. (2012). Correlation and Volatility Transmission across International Stock Markets: A Bivariate GARCH Analysis, *International Journal of Economics and Finance, 4 (3): 253-264.*

Schwert, G. W., 1989, "Why does stock market volatility change over time?", *Journal of Finance 44, 1115-1153.*

Sa Young, Lee (1998). A study of the behaviour of return volatility in the Korean Stock Market, *ProQuest Dissertations and Thesis, 1998.*

Sarkar, Chakrabarti, & Sen 2009, Indian stock market volatility in recent years: Transmission from global market, regional market and traditional domestic sectors, *Journal of Asset Management, Vol 10, 1, 63-71.*

Sen & Bandhopadhyay, (2012). On the return and volatility spillover between US and Indian Stock Market, *International Journal of Financial Management, 2(3), July*.

Singh, P, Kumar B & Pandey A (2008). Price and Volatility Spillovers across North American, European and Asian Stock Markets: With Special focus on Indian Stock Market, *Indian Institute of Management Ahmedabad, Working Paper No. 2008-12,04,*

Tripathy, T., & Gil-Alana, L.A., (2010). Suitability of volatility models for forecasting stock market returns: a study on the Indian national stock exchange. *Am. J. Appl. Sci. 7 (11),* 1487.

Wang, Y Gunasekarage, A & Power D M., 2005, "Return and Volatility spillovers from Developed to Emerging Capital Markets: The case of South Asia", *Contemporary Studies in Economic and Financial Analysis, 86, 139-166.*

Wei, W., (2002). Forecasting stock market volatility with non-linear GARCH models: a case for China. *Applied Economics. Letter 9 (3),* 163–166.

Whitelaw, R. F., (1994), Time variations and co-variations in the expectation and volatility of stock market returns, *Journal of Finance 49,* 515-541

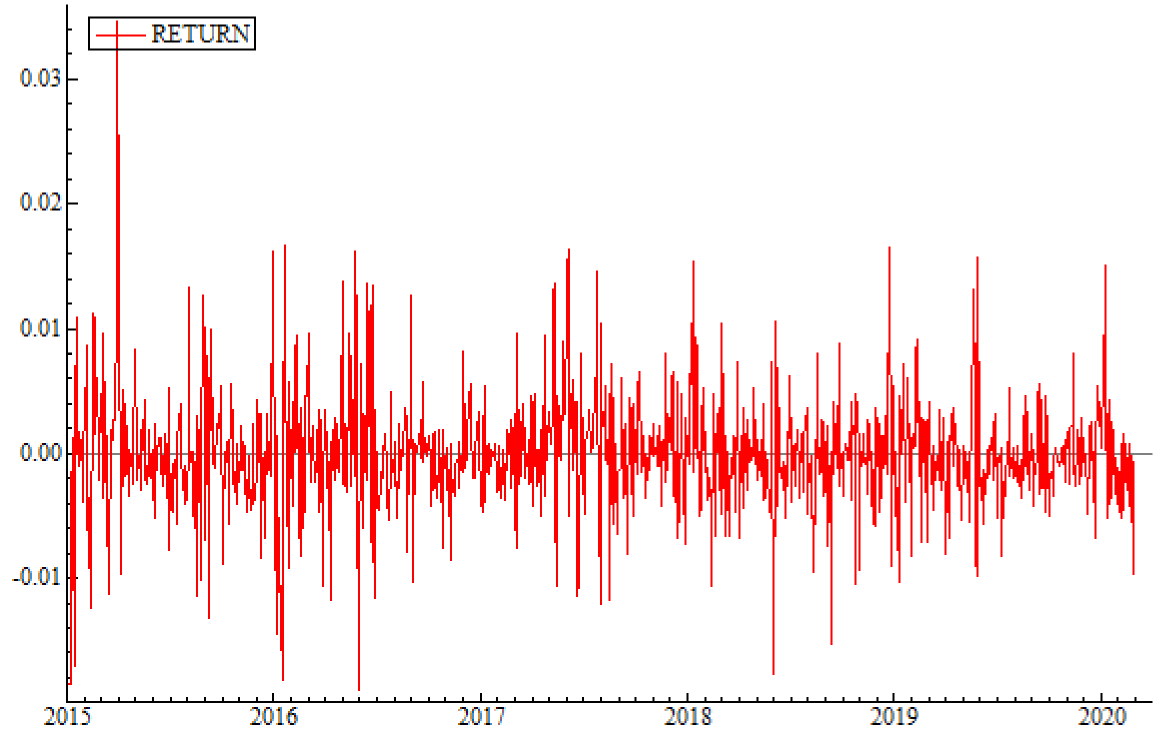
Wilhelmsson, A. (2006). GARCH forecasting performance under different distribution assumptions, Journal of Forecasting 25: 561 – 578.

Zakoian, J.M., (1994). Threshold heteroskedastic models. *J. Econ. Dyn. Control 18 (5),* 931–955.

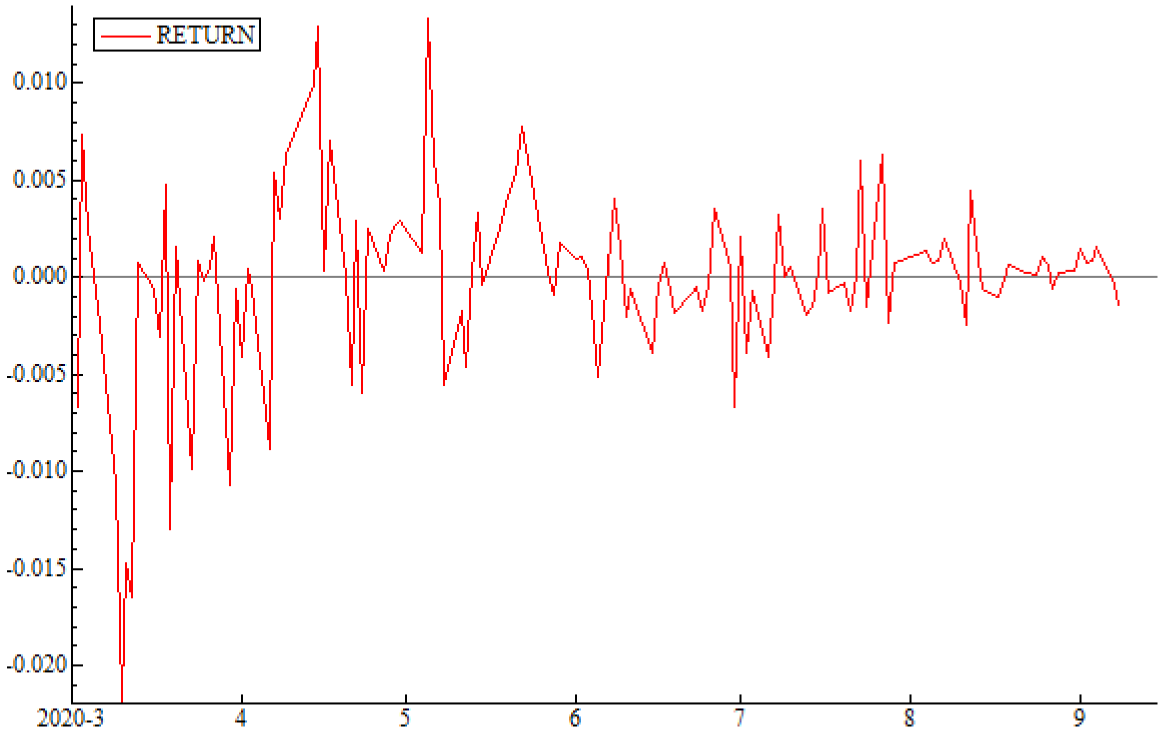
Zhou, Q., & Zhou, Z., (2005). Stock returns, volatility, and co-integration among Chinese stock markets. *China World Econ. 13,* 106–122.

Appendix

Before COVID 19



During covid 19



Before COVID 19

	RETURN
Mean	-8.78E-05
Median	-0.000257
Maximum	0.034678
Minimum	-0.018906
Std. Dev.	0.004570
Skewness	0.430902
Kurtosis	8.120876
Jarque-Bera	1434.820
Probability	0.000000
Sum	-0.112180
Sum Sq. Dev.	0.026653
Observations	1277

During COVID 19

	RETURN
Mean	-4.22E-05
Median	0.000292
Maximum	0.013326
Minimum	-0.021858
Std. Dev.	0.004797
Skewness	-1.193513
Kurtosis	7.671846
Jarque-Bera	146.7949
Probability	0.000000
Sum	-0.005404
Sum Sq. Dev.	0.002923
Observations	128

return return(-1) return(-2) return(-3)

Before COVID 19

Heteroskedasticity Test: ARCH

F-statistic	31.46684	Prob. F(1,1271)	0.0000
Obs*R-squared	30.75493	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 03/13/21 Time: 06:14
Sample (adjusted): 1/12/2015 2/28/2020
Included observations: 1273 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.57E-05	1.42E-06	11.00336	0.0000
RESID^2(-1)	0.155528	0.027726	5.609531	0.0000

R-squared	0.024159	Mean dependent var	1.85E-05
Adjusted R-squared	0.023392	S.D. dependent var	4.79E-05
S.E. of regression	4.74E-05	Akaike info criterion	-17.07498
Sum squared resid	2.85E-06	Schwarz criterion	-17.06689
Log likelihood	10870.22	Hannan-Quinn criter.	-17.07194
F-statistic	31.46684	Durbin-Watson stat	2.041210
Prob(F-statistic)	0.000000		

Null Hypothesis: RETURN has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=22)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-26.32874	0.0000
Test critical values:	1% level	-3.435263	
	5% level	-2.863597	
	10% level	-2.567915	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(RETURN)
Method: Least Squares
Date: 03/13/21 Time: 06:18
Sample (adjusted): 1/07/2015 2/28/2020
Included observations: 1276 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RETURN(-1)	-0.700029	0.026588	-26.32874	0.0000
C	-4.93E-05	0.000121	-0.406080	0.6848

R-squared	0.352380	Mean dependent var	6.96E-06
Adjusted R-squared	0.351872	S.D. dependent var	0.005382
S.E. of regression	0.004333	Akaike info criterion	-8.043484
Sum squared resid	0.023921	Schwarz criterion	-8.035409
Log likelihood	5133.743	Hannan-Quinn criter.	-8.040451
F-statistic	693.2027	Durbin-Watson stat	1.966173
Prob(F-statistic)	0.000000		

Dependent variable : RETURN
Mean Equation : ARFIMA (1, d, 1) model.
No regressor in the conditional mean
Variance Equation : GARCH (1, 1) model.
No regressor in the conditional variance
Student distribution, with 3.45062 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 5338.44
Please wait : Computing the Std Errors ...
Robust Standard Errors (Sandwich formula)

Coefficient	Std.Error	t-value	t-prob	
Cst(M)	-0.000801	0.00033173	-2.414	0.0159
d-Arfima	0.244769	0.091562	2.673	0.0076
AR(1)	0.753296	0.082817	9.096	0.0000
MA(1)	-0.846528	0.10325	-8.198	0.0000
Cst(V) x 10^6	3.597883	0.89274	4.030	0.0001
ARCH(Alpha1)	0.427037	0.081541	5.237	0.0000
GARCH(Beta1)	0.512745	0.067779	7.565	0.0000
Student(DF)	3.450616	0.35418	9.743	0.0000

No. Observations : 1277 No. Parameters : 8
Mean (Y) : -0.00009 Variance (Y) : 0.00002
Skewness (Y) : 0.43090 Kurtosis (Y) : 8.12088
Log Likelihood : 5338.443 Alpha[1]+Beta[1]: 0.93978

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 5.97471e-005
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
=> See Doornik&Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is observed.
The constraint equals -0.743707 and should be < 1.
=> See Ling &McAleer (2001) for details.

Estimated Parameters Vector :
-0.000801; 0.244769; 0.753296;-0.846528; 3.597883; 0.427037; 0.512745; 3.450621

Elapsed Time : 4.474 seconds (or 0.0745667 minutes).

GARCH(10) SPECIFICATIONS **
****

Dependent variable : RETURN SQ
Mean Equation : ARFIMA (1, d, 1) model.
No regressor in the conditional mean
Variance Equation : FIGARCH (1, d, 1) model estimated with BBM's method (Truncation order : 1000).
No regressor in the conditional variance
Student distribution, with 5.8978 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 9632.47
Please wait : Computing the Std Errors ...
Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	-0.0000154.2372e-006		-3.644	0.0003
d-Arfima	0.043496	0.20277	0.2145	0.8302
AR(1)	0.119049	0.92921	0.1281	0.8981
MA(1)	0.073740	1.1712	0.06296	0.9498
Cst(V) x 10^6	0.028707	0.0019905	14.42	0.0000
d-Figarch	0.186293	0.075977	2.452	0.0143
ARCH(Phi1)	0.146270	0.32813	0.4458	0.6558
GARCH(Beta1)	0.445072	0.035832	12.42	0.0000
Student(DF)	5.897802	0.40516	14.56	0.0000

No. Observations : 1278 No. Parameters : 9
Mean (Y) : 0.00002 Variance (Y) : 0.00000
Skewness (Y) : 9.59718 Kurtosis (Y) : 169.24446
Log Likelihood : 9632.471

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the FIGARCH (1,d,1) is
not observed (0.258779<0.14627<-0.0485448<-0.0500746 not valid).
=> See Bolle

rslev and Mikkelsen (1996) for more details.
WARNING ! There are 1 estimated negative conditional variances.

Estimated Parameters Vector :
-0.000015; 0.043496; 0.119049; 0.073740; 0.028707; 0.186293; 0.146270; 0.445072; 5.897807

Elapsed Time : 14.703 seconds (or 0.24505 minutes).

Starting estimation process...

During COVID 19

Heteroskedasticity Test: ARCH

F-statistic	4.496360	Prob. F(1,122)	0.0360
Obs*R-squared	4.407626	Prob. Chi-Square(1)	0.0358

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 03/13/21 Time: 03:52
Sample (adjusted): 3/09/2020 9/08/2020
Included observations: 124 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.46E-05	3.90E-06	3.739993	0.0003
RESID^2(-1)	0.188569	0.088928	2.120462	0.0360

R-squared	0.035545	Mean dependent var	1.80E-05
Adjusted R-squared	0.027640	S.D. dependent var	4.02E-05
S.E. of regression	3.96E-05	Akaike info criterion	-17.41915
Sum squared resid	1.91E-07	Schwarz criterion	-17.37366
Log likelihood	1081.987	Hannan-Quinn criter.	-17.40067
F-statistic	4.496360	Durbin-Watson stat	1.964286
Prob(F-statistic)	0.035992		

Null Hypothesis: RETURN has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-4.701579	0.0002
Test critical values:	1% level	-3.482879	
	5% level	-2.884477	
	10% level	-2.579080	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(RETURN)
Method: Least Squares
Date: 03/13/21 Time: 03:55
Sample (adjusted): 3/05/2020 9/08/2020
Included observations: 126 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RETURN(-1)	-0.457913	0.097396	-4.701579	0.0000
D(RETURN(-1))	-0.300588	0.085142	-3.530456	0.0006
C	-9.18E-05	0.000384	-0.239290	0.8113

R-squared	0.388354	Mean dependent var	-3.25E-05
Adjusted R-squared	0.378409	S.D. dependent var	0.005463
S.E. of regression	0.004307	Akaike info criterion	-8.033591
Sum squared resid	0.002282	Schwarz criterion	-7.966060
Log likelihood	509.1162	Hannan-Quinn criter.	-8.006155
F-statistic	39.04837	Durbin-Watson stat	1.915265
Prob(F-statistic)	0.000000		

Null Hypothesis: RETURN is stationary
Exogenous: Constant
Bandwidth: 5 (Newey-West automatic) using Bartlett kernel

		LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic		0.234114
Asymptotic critical values*:	1% level	0.739000
	5% level	0.463000
	10% level	0.347000

*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)

Residual variance (no correction)	2.28E-05
HAC corrected variance (Bartlett kernel)	4.92E-05

KPSS Test Equation
Dependent Variable: RETURN
Method: Least Squares
Date: 03/13/21 Time: 03:56
Sample (adjusted): 3/03/2020 9/08/2020
Included observations: 128 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-4.22E-05	0.000424	-0.099566	0.9208

R-squared	0.000000	Mean dependent var	-4.22E-05
Adjusted R-squared	0.000000	S.D. dependent var	0.004797
S.E. of regression	0.004797	Akaike info criterion	-7.833739
Sum squared resid	0.002923	Schwarz criterion	-7.811457
Log likelihood	502.3593	Hannan-Quinn criter.	-7.824686
Durbin-Watson stat	1.283860		

Dependent variable : RETURN
Mean Equation : ARFIMA (1, d, 1) model.
No regressor in the conditional mean
Variance Equation : GARCH (1, 1) model.
No regressor in the conditional variance

Student distribution, with 3.66253 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 549.743
Please wait : Computing the Std Errors ...
Robust Standard Errors (Sandwich formula)
Coefficient Std.Error t-value t-prob
Cst(M) 0.000327 0.00093546 0.3491 0.7277
d-Arfima 0.113413 0.61794 0.1835 0.8547
AR(1) -0.454500 0.45659 -0.9954 0.3215
MA(1) 0.407459 0.52637 0.7741 0.4404
Cst(V) x 10^6 0.000000 8.3739 0.00 1.0000
ARCH(Alpha1) 0.091305 4.6900 0.01947 0.9845
GARCH(Beta1) 0.902818 3.9001 0.2315 0.8173
Student(DF) 3.662531 24.032 0.1524 0.8791

No. Observations : 129 No. Parameters : 8
Mean (Y) : -0.00009 Variance (Y) : 0.00002
Skewness (Y) : -1.16016 Kurtosis (Y) : 7.48380
Log Likelihood : 549.743 Alpha[1]+Beta[1]: 0.99412

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance does not exist and/or is not positive.
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
=> See Doornik&Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is observed.
The constraint equals 0.856734 and should be < 1.
=> See Ling &McAleer (2001) for details.

ependent variable : RETURN SQ
Mean Equation : ARFIMA (1, d, 1) model.
No regressor in the conditional mean
Variance Equation : FIGARCH (1, d, 1) model estimated with BBM's method (Truncation order : 1000).
No regressor in the conditional variance
Student distribution, with 5.70176 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 1200.73
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)
Coefficient Std.Error t-value t-prob
Cst(M) -0.0000041.5948e-005 -0.2526 0.8010
d-Arfima 0.165876 0.26883 0.6170 0.5384
AR(1) 0.149455 2.2169 0.06742 0.9464
MA(1) 0.135941 1.3382 0.1016 0.9193
Cst(V) x 10^6 0.0000096.3556e-006 1.478 0.1422

d-Figarch 0.629764 0.28047 2.245 0.0266
ARCH(Phi1) 0.260656 0.32488 0.8023 0.4240
GARCH(Beta1) 0.228739 0.33329 0.6863 0.4938
Student(DF) 5.701763 2.4174 2.359 0.0200

No. Observations : 129 No. Parameters : 9
Mean (Y) : 0.00002 Variance (Y) : 0.00000
Skewness (Y) : 4.84692 Kurtosis (Y) : 31.84624
Log Likelihood : 1200.727

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the FIGARCH (1,d,1) is
observed (-0.401025<0.260656<0.456745 and 0.0475711<0.151352 valid).
=> See Bollerslev and Mikkelsen (1996) for more details.

Estimated Parameters Vector :
-0.000004; 0.165876; 0.149455; 0.135941; 0.000009; 0.629764; 0.260656; 0.228739; 5.701768

Elapsed Time : 0.74 seconds (or 0.0123333 minutes).

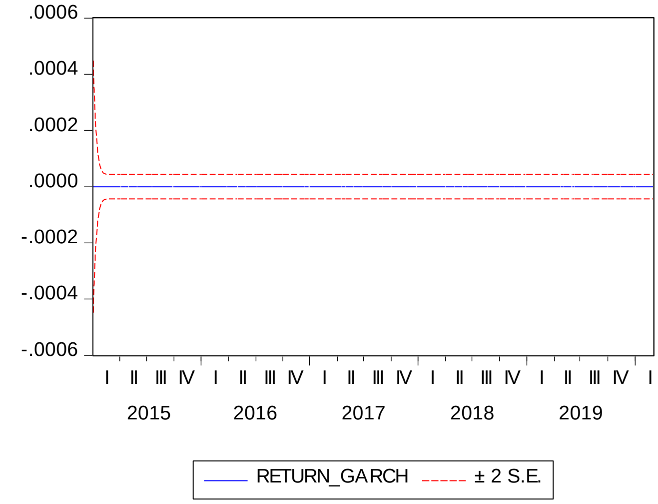
Starting estimation process...
Warning: Low initial temperature, may not leave sufficient room
for exploring the full parameter space
Missing found in MaxSA
Runtime error occurred in MaxSA(242), call trace:
/Applications/OxMetrics6/ox/packages/Garch6/maxsa.ox (242): MaxSA
/Applications/OxMetrics6/ox/packages/Garch6/procs_garch/garch_oxpack_oxmetrics5.ox (174): Estimate
Runtime error: in callback function

BEFORE COVID 19
Dependent Variable: RETURN_SQ
Method: ML ARCH - Student's t distribution (BFGS / Marquardt steps)
Date: 03/13/21 Time: 08:48
Sample (adjusted): 1/06/2015 2/28/2020
Included observations: 1277 after adjustments
Convergence achieved after 87 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
t-distribution degree of freedom parameter fixed at 10
GARCH = C(1) + C(2)*RESID(-1)^2 + C(3)*GARCH(-1)

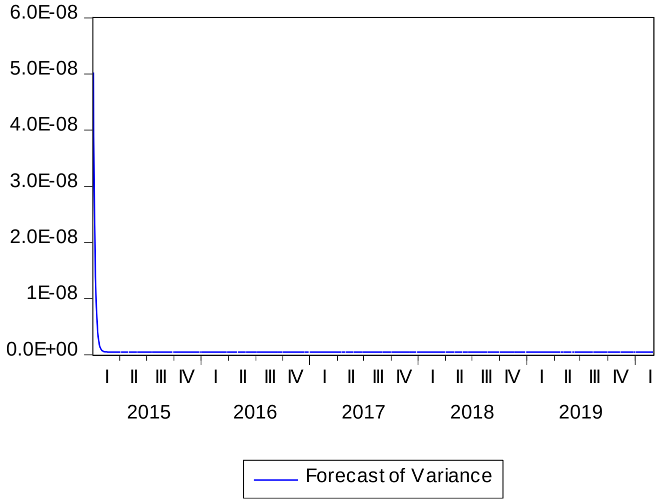
Variable	Coefficient	Std. Error	z-Statistic	Prob.
		Variance Equation		
C	1.09E-10	5.56E-12	19.58677	0.0000
RESID(-1)^2	0.422818	0.032713	12.92527	0.0000

GARCH(-1)	0.346635	0.017764	19.51287	0.0000

R-squared	-0.141164	Mean dependent var	2.09E-05
Adjusted R-squared	-0.140270	S.D. dependent var	5.56E-05
S.E. of regression	5.94E-05	Akaike info criterion	-18.19526
Sum squared resid	4.50E-06	Schwarz criterion	-18.18315
Log likelihood	11620.67	Hannan-Quinn criter.	-18.19071
Durbin-Watson stat	1.182267		



Forecast: RETURN_GARCH	
Actual: RETURN_SQ	
Forecast sample: 1/05/2015 2/28/2020	
Adjusted sample: 1/06/2015 2/28/2020	
Included observations: 1277	
Root Mean Squared Error	5.94E-05
Mean Absolute Error	2.09E-05
Mean Abs. Percent Error	NA
Theil Inequality Coefficient	1.000000
Bias Proportion	0.123702
Variance Proportion	NA
Covariance Proportion	NA
Theil U2 Coefficient	NA
Symmetric MAPE	NA

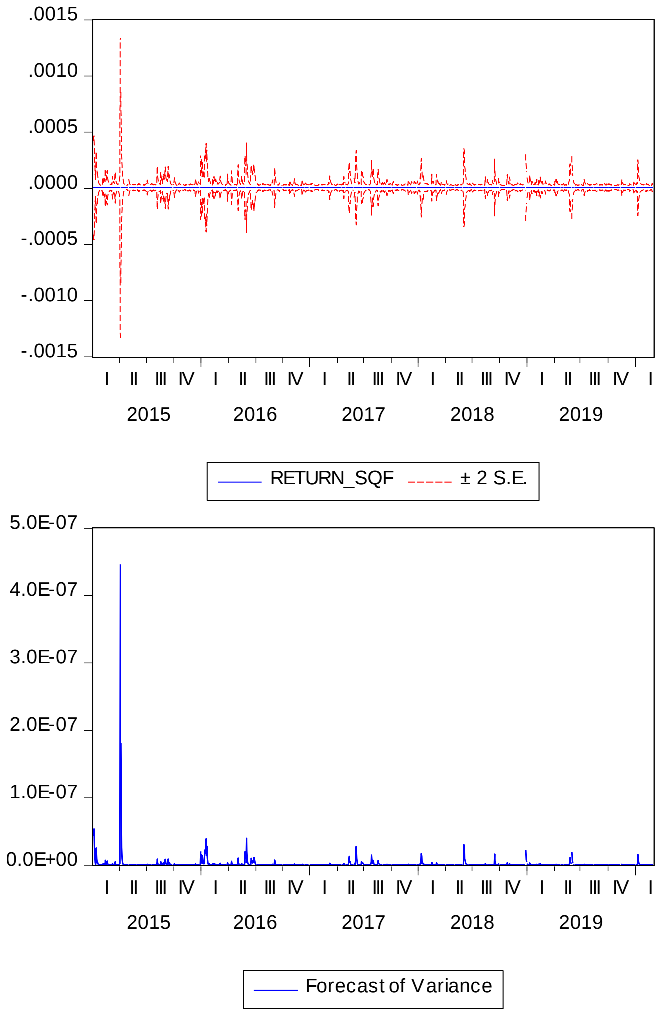


Dependent Variable: RETURN_SQ
Method: ML ARCH - Student's t distribution (BFGS / Marquardt steps)
Date: 03/13/21 Time: 08:57
Sample (adjusted): 1/06/2015 2/28/2020
Included observations: 1277 after adjustments
Convergence not achieved after 500 iterations
Coefficient covariance computed using outer product of gradients

Presample variance: backcast (parameter = 0.7)
t-distribution degree of freedom parameter fixed at 10
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-1)^2*(RESID(-1)<0) +
C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
LOG(GARCH)	-3.09E-07	2.55E-08	-12.13351	0.0000
Variance Equation				
C	1.44E-10	8.17E-12	17.57897	0.0000
RESID(-1)^2	0.311155	0.026713	11.64821	0.0000
RESID(-1)^2*(RESID(-1)<0)	-2.278255	0.372920	-6.109238	0.0000
GARCH(-1)	0.354659	0.018083	19.61272	0.0000

R-squared	-0.069635	Mean dependent var	2.09E-05
Adjusted R-squared	-0.069635	S.D. dependent var	5.56E-05
S.E. of regression	5.75E-05	Akaike info criterion	-18.43700
Sum squared resid	4.22E-06	Schwarz criterion	-18.41682
Log likelihood	11777.02	Hannan-Quinn criter.	-18.42942
Durbin-Watson stat	1.257412		



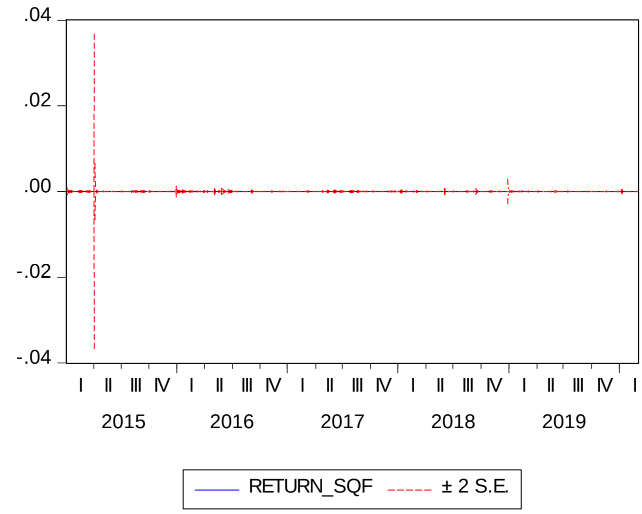
Forecast: RETURN_SQF	
Actual: RETURN_SQ	
Forecast sample: 1/05/2015 2/28/2020	
Adjusted sample: 1/06/2015 2/28/2020	
Included observations: 1277	
Root Mean Squared Error	5.75E-05
Mean Absolute Error	2.01E-05
Mean Abs. Percent Error	NA
Theil Inequality Coefficient	0.869122
Bias Proportion	0.060415
Variance Proportion	0.921490
Covariance Proportion	0.018095
Theil U2 Coefficient	NA
Symmetric MAPE	117.6532

Dependent Variable: RETURN_SQ
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 03/13/21 Time: 09:03
Sample (adjusted): 1/06/2015 2/28/2020
Included observations: 1277 after adjustments
Convergence achieved after 48 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(1) + C(2)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(3)
*RESID(-1)/@SQRT(GARCH(-1)) + C(4)*LOG(GARCH(-1))

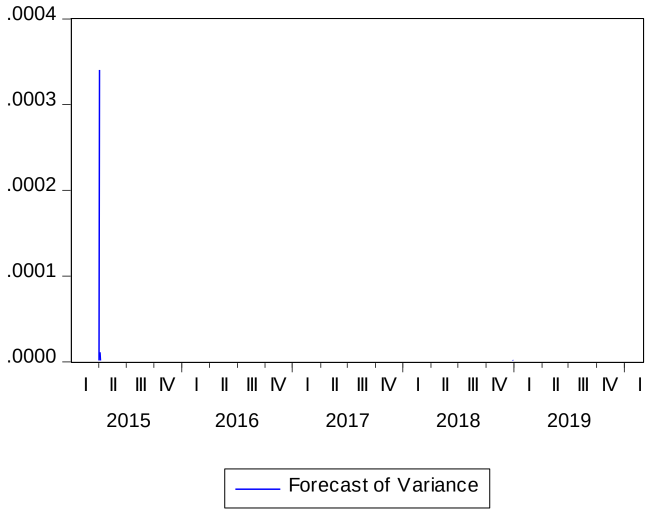
Variable	Coefficient	Std. Error	z-Statistic	Prob.
		Variance Equation		
C(1)	-5.489291	0.211446	-25.96078	0.0000

C(2)	3.137948	3.807527	0.824143	0.4099
C(3)	-2.350636	3.801991	-0.618264	0.5364
C(4)	0.743315	0.010146	73.26305	0.0000

R-squared	-0.141164	Mean dependent var	2.09E-05
Adjusted R-squared	-0.140270	S.D. dependent var	5.56E-05
S.E. of regression	5.94E-05	Akaike info criterion	-17.30996
Sum squared resid	4.50E-06	Schwarz criterion	-17.29382
Log likelihood	11056.41	Hannan-Quinn criter.	-17.30390
Durbin-Watson stat	1.182267		



Forecast: RETURN_SQF	
Actual: RETURN_SQ	
Forecast sample: 1/05/2015 2/28/2020	
Adjusted sample: 1/06/2015 2/28/2020	
Included observations: 1277	
Root Mean Squared Error	5.94E-05
Mean Absolute Error	2.09E-05
Mean Abs. Percent Error	NA
Theil Inequality Coefficient	1.000000
Bias Proportion	0.123702
Variance Proportion	NA
Covariance Proportion	NA
Theil U2 Coefficient	NA
Symmetric MAPE	NA



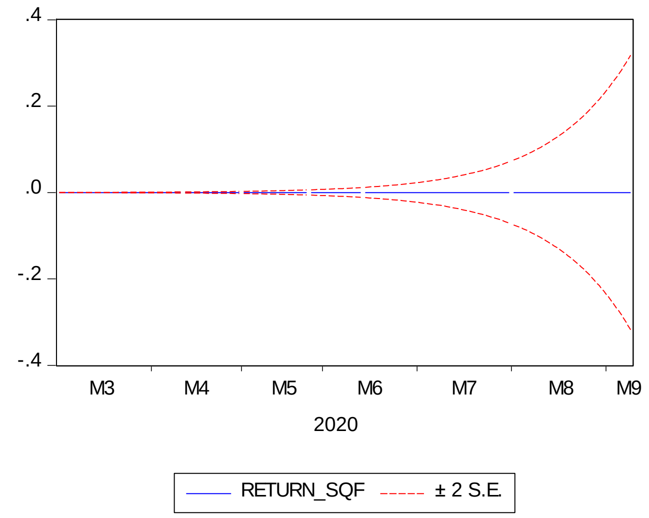
DURING COVID 19

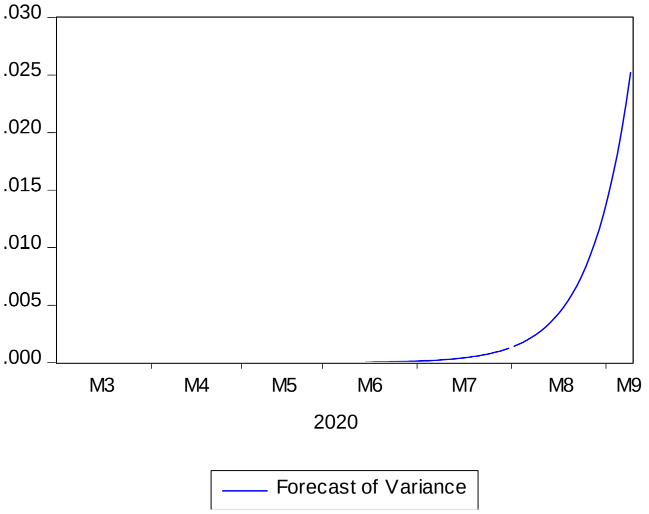
Dependent Variable: RETURN_SQ
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 03/13/21 Time: 09:10

Sample (adjusted): 3/03/2020 9/08/2020
Included observations: 128 after adjustments
Convergence not achieved after 500 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(1) + C(2)*RESID(-1)^2 + C(3)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
	Variance Equation			
C	-9.62E-13	8.98E-13	-1.070545	0.2844
RESID(-1)^2	0.306484	0.072213	4.244165	0.0000
GARCH(-1)	0.810981	0.017539	46.23952	0.0000

R-squared	-0.148958	Mean dependent var	2.28E-05
Adjusted R-squared	-0.139982	S.D. dependent var	5.94E-05
S.E. of regression	6.34E-05	Akaike info criterion	-17.91851
Sum squared resid	5.15E-07	Schwarz criterion	-17.85166
Log likelihood	1149.785	Hannan-Quinn criter.	-17.89135
Durbin-Watson stat	0.985724		





Forecast: RETURN_SQF	
Actual: RETURN_SQ	
Forecast sample: 3/02/2020 9/08/2020	
Adjusted sample: 3/03/2020 9/08/2020	
Included observations: 128	
Root Mean Squared Error	6.34E-05
Mean Absolute Error	2.28E-05
Mean Abs. Percent Error	100.0000
Theil Inequality Coefficient	1.000000
Bias Proportion	0.129646
Variance Proportion	NA
Covariance Proportion	NA
Theil U2 Coefficient	1.000009
Symmetric MAPE	200.0000

Dependent Variable: RETURN_SQ
Method: ML ARCH - Generalized error distribution (GED) (BFGS / Marquardt
steps)
Date: 03/13/21 Time: 09:13
Sample (adjusted): 3/03/2020 9/08/2020
Included observations: 128 after adjustments
Convergence not achieved after 500 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GED parameter fixed at 1.5
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-1)^2*(RESID(-1)<0) +

C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.

LOG(GARCH)	-9.86E-08	2.09E-08	-4.717335	0.0000
Variance Equation				
C	8.88E-13	1.74E-12	0.511436	0.6090
RESID(-1)^2	0.019213	0.007688	2.499106	0.0125
RESID(-1)^2*(RESID(-1)<0)	-0.463490	0.572298	-0.809876	0.4180
GARCH(-1)	0.917833	0.006971	131.6718	0.0000

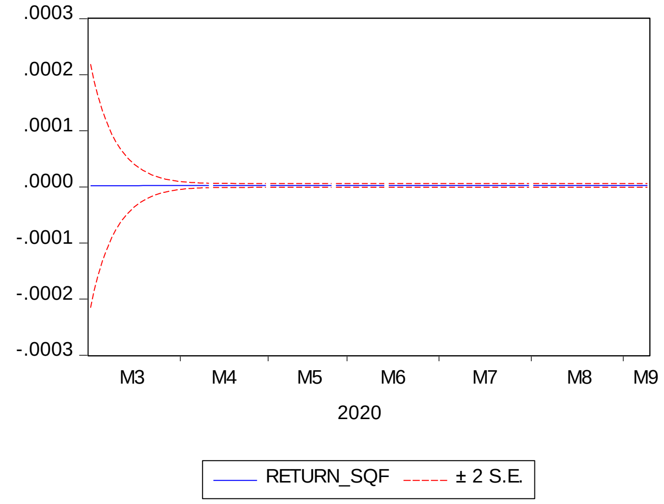
R-squared	-0.125216	Mean dependent var	2.28E-05
Adjusted R-squared	-0.125216	S.D. dependent var	5.94E-05
S.E. of regression	6.30E-05	Akaike info criterion	-18.75171
Sum squared resid	5.04E-07	Schwarz criterion	-18.64030
Log likelihood	1205.109	Hannan-Quinn criter.	-18.70644
Durbin-Watson stat	1.006413		

Dependent Variable: RETURN_SQ
Method: ML ARCH - Generalized error distribution (GED) (BFGS / Marquardt
steps)
Date: 03/13/21 Time: 09:13
Sample (adjusted): 3/03/2020 9/08/2020
Included observations: 128 after adjustments
Convergence not achieved after 500 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GED parameter fixed at 1.5
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-1)^2*(RESID(-1)<0) +
C(5)*GARCH(-1)

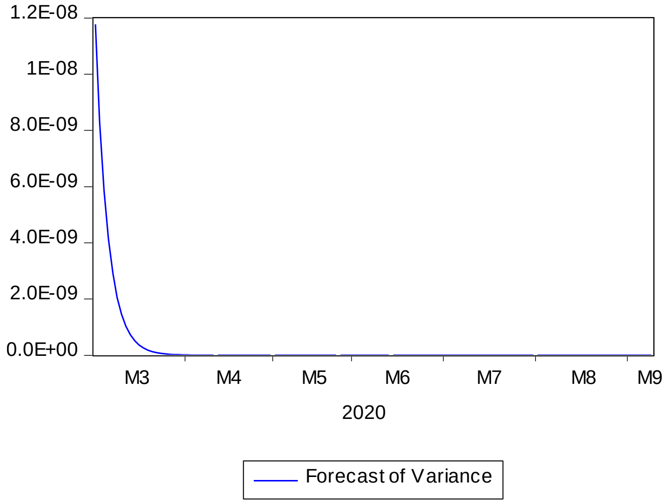
Variable	Coefficient	Std. Error	z-Statistic	Prob.
LOG(GARCH)	-9.86E-08	2.09E-08	-4.717335	0.0000
Variance Equation				
C	8.88E-13	1.74E-12	0.511436	0.6090
RESID(-1)^2	0.019213	0.007688	2.499106	0.0125
RESID(-1)^2*(RESID(-1)<0)	-0.463490	0.572298	-0.809876	0.4180
GARCH(-1)	0.917833	0.006971	131.6718	0.0000

R-squared	-0.125216	Mean dependent var	2.28E-05
Adjusted R-squared	-0.125216	S.D. dependent var	5.94E-05
S.E. of regression	6.30E-05	Akaike info criterion	-18.75171
Sum squared resid	5.04E-07	Schwarz criterion	-18.64030
Log likelihood	1205.109	Hannan-Quinn criter.	-18.70644

Durbin-Watson stat	1.006413



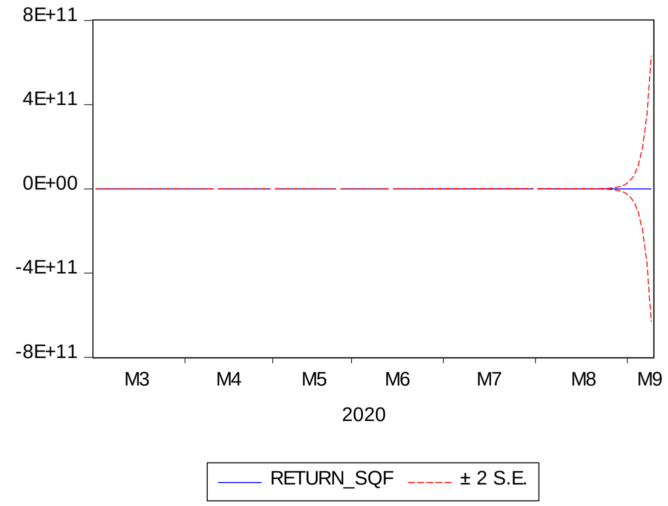
Forecast: RETURN_SQF	
Actual: RETURN_SQ	
Forecast sample: 3/02/2020 9/08/2020	
Adjusted sample: 3/03/2020 9/08/2020	
Included observations: 128	
Root Mean Squared Error	6.26E-05
Mean Absolute Error	2.23E-05
Mean Abs. Percent Error	16505.97
Theil Inequality Coefficient	0.949564
Bias Proportion	0.105081
Variance Proportion	0.886549
Covariance Proportion	0.008370
Theil U2 Coefficient	0.925089
Symmetric MAPE	131.3196



Dependent Variable: RETURN_SQ
Method: ML ARCH - Generalized error distribution (GED) (BFGS / Marquardt
steps)
Date: 03/13/21 Time: 09:24
Sample (adjusted): 3/03/2020 9/08/2020
Included observations: 128 after adjustments
Failure to improve likelihood (singular hessian) after 116 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(1) + C(2)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(3)
*RESID(-1)/@SQRT(GARCH(-1)) + C(4)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
Variance Equation				
C(1)	0.289558	0.067125	4.313695	0.0000
C(2)	0.204161	2.128395	0.095923	0.9236
C(3)	-0.320431	2.219646	-0.144361	0.8852
C(4)	1.014333	1.7E-104	5.8E+103	0.0000
GED PARAMETER	0.267647	0.037376	7.161021	0.0000

R-squared	-0.148958	Mean dependent var	2.28E-05
Adjusted R-squared	-0.139982	S.D. dependent var	5.94E-05
S.E. of regression	6.34E-05	Akaike info criterion	-20.11913
Sum squared resid	5.15E-07	Schwarz criterion	-20.00772
Log likelihood	1292.624	Hannan-Quinn criter.	-20.07387
Durbin-Watson stat	0.985724		



Forecast: RETURN_SQF	
Actual: RETURN_SQ	
Forecast sample: 3/02/2020 9/08/2020	
Adjusted sample: 3/03/2020 9/08/2020	
Included observations: 128	
Root Mean Squared Error	6.34E-05
Mean Absolute Error	2.28E-05
Mean Abs. Percent Error	100.0000
Theil Inequality Coefficient	1.000000
Bias Proportion	0.129646
Variance Proportion	NA
Covariance Proportion	NA
Theil U2 Coefficient	1.000009
Symmetric MAPE	200.0000