
Mathematical Analysis of Natural Convection Heat Transfer in Nanofluids within a Vertical Annulus under Thermal Radiation Effects

¹*MKPEDEM K. A; ²Nyah R; ³FADEPO J. T

1. *Department of Mathematics, University of Abuja, Nigeria.

2. Faculty of Chemical and Process Engineering, Universiti Malaysia Pahang Al-Sultan Abdullah.

3. Department of Industrial Mathematics, Federal University of Science and Technology, Minna, Niger State, Nigeria.

Corresponding author: *kufremkpedem@gmail.com, olayiwola.rasaq@Futminna.edu.ng, fadepoj@gmail.com

Abstract

This study presents a mathematical analysis of convective heat transfer mechanisms relevant to nanofluid flow systems, with particular emphasis on applications in vertical annular geometries subjected to thermal radiation effects. The work examines the fundamental principles of forced convection, dimensional analysis, Reynolds analogy, and heat exchanger effectiveness as essential tools for understanding thermal transport phenomena in nanofluids. Dimensionless parameters including the Reynolds number, Prandtl number, Nusselt number, Stanton number, and Number of Transfer Units (NTU) are derived and analyzed to establish generalized heat-transfer correlations. The influence of nanoparticle concentration on effective thermophysical properties and subsequent heat-transfer enhancement is discussed through modified Reynolds analogy and heat exchanger formulations. The study demonstrates how nanoparticle dispersion improves thermal conductivity, convective heat-transfer coefficients, and overall thermal performance. Furthermore, the effectiveness-NTU method is employed to evaluate heat exchanger performance under varying operating conditions. The developed mathematical framework provides a foundation for predicting and optimizing heat transfer in nanofluid-filled annular systems and advanced thermal engineering applications.

Keywords: Forced Convection, Nanofluids, Reynolds Analogy, Dimensional Analysis, Heat Exchanger Effectiveness, Heat Transfer Enhancement.

1. Forced Convection

Forced convection refers to the transfer of thermal energy between a solid surface and a moving fluid when the fluid motion is induced by an external mechanism such as a pump, fan, compressor, or pressure gradient. Unlike natural convection, where fluid motion arises from buoyancy forces generated by temperature differences, forced convection is characterized by externally driven flow, resulting in enhanced heat transfer rates and improved thermal performance. The mechanism plays a fundamental role in engineering applications such as heat exchangers, cooling systems, chemical reactors, solar collectors, electronic devices, and thermal management equipment (Karwa, 2017; Hazeri-Mahmel, Shekari, & Tayebi, 2020).

The rate of heat transfer in forced convection is commonly described by Newton's law of cooling

$$Q = hA(T_w - T_f) \quad (1)$$

where Q denotes the heat transfer rate, h is the convective heat transfer coefficient, A represents the heat transfer surface area, T_w is the wall temperature, and T_f is the bulk fluid temperature. The determination of the convective heat transfer coefficient constitutes the primary challenge in forced convection analysis because it depends on fluid properties, flow characteristics, and surface geometry. Recent studies have demonstrated that nanofluids significantly enhance the convective heat transfer coefficient due to improved thermal conductivity and particle-induced energy transport mechanisms (Sundar, Sharma, & Singh, 2016; Rashad, 2018). Fluid motion in forced convection systems is generally characterized using the Reynolds number,

$$Re = \frac{\rho UL}{\mu}$$

where ρ is the fluid density, U is the characteristic velocity, L is the characteristic length, and μ is the dynamic viscosity. The Reynolds number measures the ratio of inertial forces to viscous forces and determines whether the flow is laminar or turbulent. For internal flow in circular ducts, laminar flow typically occurs when $Re < 2300$, whereas turbulent flow develops for $Re > 4000$. Between these limits, the flow remains in a transitional state. The hydrodynamic behaviour strongly influences thermal boundary layer development and consequently affects the heat transfer process (Incropera et al., 2017; Cengel & Ghajar, 2020).

In convective heat transfer analyses, dimensionless parameters provide a convenient means of correlating experimental and theoretical results. Among these parameters, the Nusselt number, Prandtl number, and Reynolds number are of particular importance. The Nusselt number represents the ratio of convective to conductive heat transfer, while the Prandtl number characterizes the relative thicknesses of momentum and thermal boundary layers. Their relationship is commonly expressed as

$$Nu = f(Re, Pr) \quad (2)$$

which forms the basis of numerous empirical and semi-empirical heat transfer correlations. For nanofluid flows, additional parameters such as nanoparticle volume fraction, particle diameter, and Brownian motion effects may also influence the heat transfer behaviour (Sheikholeslami, 2018; Mahian et al., 2019).

The enhancement of forced convection heat transfer through nanofluids has attracted considerable attention over the last decade. Experimental and numerical investigations have shown that the addition of metallic and metal-oxide nanoparticles to conventional base fluids significantly increases thermal conductivity and heat transfer efficiency. For example, Al_2O_3 -water, CuO – water, TiO_2 -water, and hybrid nanofluids have demonstrated superior thermal performance compared with ordinary fluids under identical operating conditions. Such enhancements are attributed to increased thermal diffusivity, particle-fluid interactions, Brownian motion, and micro-convection effects within the fluid medium (Sundar et al., 2016; Xuan, Li, & Yu, 2018; Mahian et al., 2019).

Figure 1 illustrates the basic mechanism of forced convection heat transfer in which fluid motion generated by an external driving force transports thermal energy away from a heated surface. The resulting thermal boundary layer develops along the flow direction, and heat is continuously transferred from the solid surface to the moving fluid.

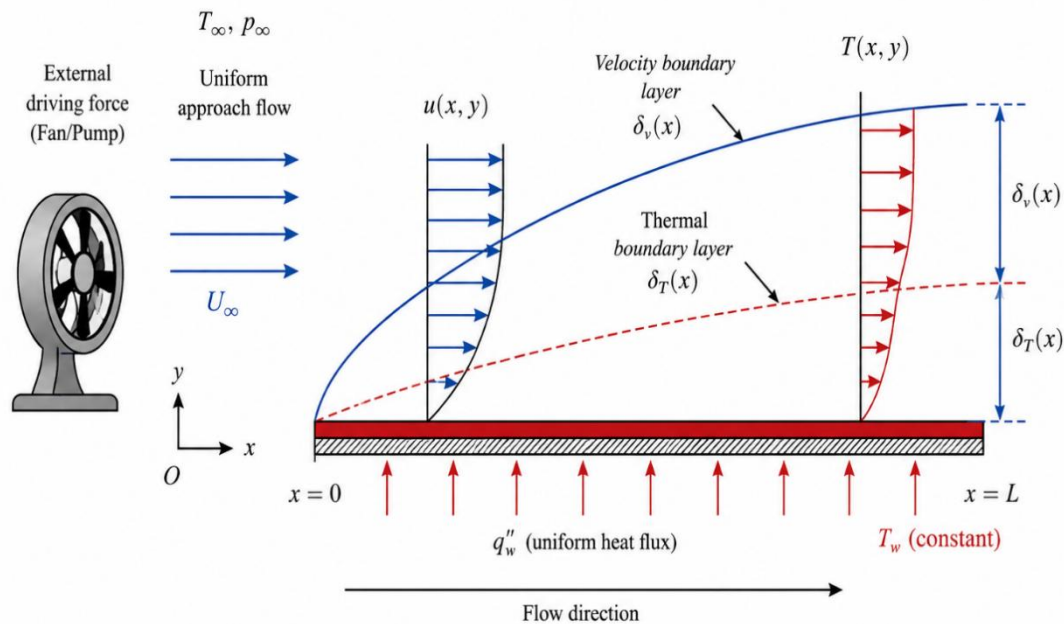


Figure 1. Schematic representation of forced convection heat transfer over a heated surface.

Although the primary focus of the present study is natural convection within a vertical annulus, understanding forced convection principles is essential because many of the dimensionless parameters, transport mechanisms, and heat transfer correlations employed in natural convection analysis originate from forced convection theory. Furthermore, the mathematical framework developed for forced convection provides a useful foundation for examining buoyancy-driven nanofluid flows subjected to thermal radiation effects.

2. Dimensional Analysis

Dimensional analysis is a powerful mathematical technique widely employed in heat transfer and fluid mechanics to establish functional relationships among governing physical variables. The method reduces the complexity of a physical problem by transforming dimensional quantities into a set of dimensionless parameters that characterize the transport process. In convective heat transfer studies, dimensional analysis provides the theoretical foundation for developing generalized heat-transfer correlations applicable to a wide range of operating conditions (Karwa, 2017; Cengel & Ghajar, 2020).

For forced convection heat transfer in nanofluids flowing through a vertical annulus, the convective heat transfer coefficient, h , depends on several physical variables, including the fluid density ρ , dynamic viscosity μ , thermal conductivity k , specific heat capacity c_p , mean flow velocity U , characteristic length L , and temperature difference ΔT . Therefore, the heat transfer coefficient may be expressed as

$$h = f(\rho, \mu, k, c_p, U, L, \Delta T) \quad (3)$$

where f denotes an unknown functional relationship.

According to the Buckingham π theorem, a physical problem involving n variables and m fundamental dimensions can be represented by $(n - m)$ independent dimensionless groups. Applying this theorem to the present convection problem yields several important dimensionless parameters that govern heat transfer behaviour. These parameters provide a concise representation of the combined

effects of fluid properties, geometry, and flow conditions on thermal transport (White, 2016; Incropera et al., 2017).

The first dimensionless parameter is the Reynolds number, which measures the relative importance of inertial forces compared with viscous forces:

$$Re = \frac{\rho UL}{\mu}$$

where:

Re = Reynolds number, **ρ** = fluid density, **U** = characteristic fluid velocity, **L** = characteristic length, **μ** = dynamic viscosity of the fluid. A large Reynolds number indicates dominance of inertial forces and promotes turbulence, whereas smaller values correspond to laminar flow conditions. The Reynolds number therefore plays a fundamental role in determining flow structure and heat-transfer characteristics.

The second important dimensionless group is the Prandtl number, defined as

$$Pr = \frac{c_p \mu}{k}$$

Where,

Pr = Prandtl number, **c_p** = specific heat capacity at constant pressure, **μ** = dynamic viscosity, **k** = thermal conductivity. The Prandtl number represents the ratio of momentum diffusivity to thermal diffusivity. Fluids with large Prandtl numbers possess relatively thin thermal boundary layers, while fluids with low Prandtl numbers exhibit more rapid thermal diffusion. For nanofluids, the effective thermal conductivity and viscosity introduced by suspended nanoparticles directly influence the magnitude of the Prandtl number and consequently the heat-transfer process (Mahian et al., 2019).

The third dimensionless parameter is the Nusselt number, which expresses the ratio of convective heat transfer to pure conductive heat transfer:

$$Nu = \frac{hL}{k}$$

The Nusselt number serves as the primary indicator of convective heat-transfer performance. Higher values of Nu correspond to stronger convection and greater thermal transport efficiency.

Combining these dimensionless parameters leads to the general functional relationship

$$Nu = F(Re, Pr) \quad (4)$$

which forms the basis of most empirical and semi-empirical heat-transfer correlations. For nanofluid systems, additional parameters such as nanoparticle volume fraction ϕ , particle diameter dp , thermal radiation parameter Rd , and annulus aspect ratio may also influence the heat-transfer characteristics. Consequently, the generalized correlation may be expressed as

$$Nu = F(Re, Pr, \phi, Rd) \quad (5)$$

where ϕ denotes nanoparticle concentration and Rd represents the radiation parameter.

Figure 2 illustrates the transformation of dimensional variables into the principal dimensionless groups governing convective heat transfer. The figure highlights the relationship between fluid properties, flow conditions, and the resulting Nusselt number correlation.

Physical Variables

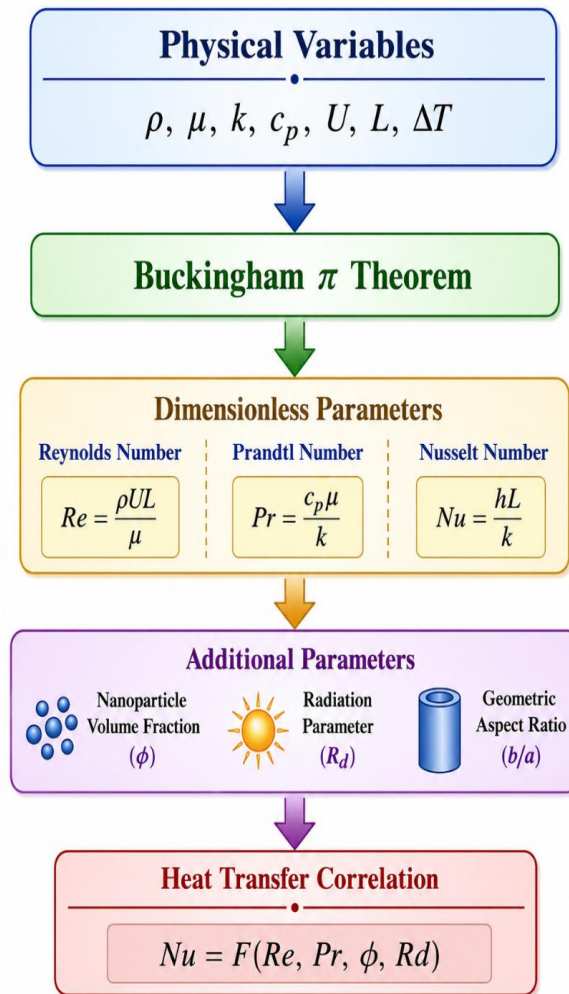


Figure 2. Schematic representation of dimensional analysis leading to heat-transfer correlations.

The application of dimensional analysis significantly reduces experimental effort and facilitates the development of generalized predictive models. Furthermore, dimensionless correlations provide a convenient framework for comparing heat-transfer performance among different nanofluids, geometrical configurations, and operating conditions. Consequently, dimensional analysis remains an indispensable tool in the theoretical and experimental investigation of convection heat transfer in vertical annular systems subjected to thermal radiation effects (Mahian et al., 2019; Sheikholeslami, 2018; Xuan, Li, & Yu, 2018).

3. Reynolds Analogy

Reynolds analogy is one of the fundamental concepts in convective heat transfer theory, providing a relationship between momentum transfer and heat transfer in fluid flow systems. The analogy was originally developed by Osborne Reynolds and is based on the observation that both momentum and thermal energy are transported by similar mechanisms within a flowing fluid. Consequently, under certain conditions, the mathematical equations governing momentum transport and heat transport exhibit analogous forms. This relationship has been widely employed in the analysis of heat exchangers, pipe flows, boundary-layer problems, and thermal systems involving forced convection (Cengel & Ghajar, 2020; Incropera et al., 2017).

The Reynolds analogy is particularly applicable to turbulent flow conditions where momentum and thermal energy are transported predominantly by eddy diffusion. For fluids having a Prandtl number close to unity ($Pr \approx 1$), the velocity and thermal boundary layers develop in a similar manner, thereby allowing a direct relationship between wall shear stress and heat transfer rate. This simplification has

made Reynolds analogy an important tool for estimating convective heat-transfer coefficients from friction-factor measurements (White, 2016; Karwa, 2017).

3.1 Mathematical Development

Consider a fluid flowing over a heated surface with free-stream velocity U_∞ . The wall shear stress is given by

$$\tau_w = \mu \left. \frac{du}{dy} \right|_w \quad (6)$$

where

τ_w = wall shear stress, μ = dynamic viscosity, u = fluid velocity, y = coordinate normal to the wall.

The local skin-friction coefficient is defined as

$$C_f = \frac{\tau_w}{(\frac{1}{2}\rho U_\infty^2)} \quad (7)$$

where ρ denotes the fluid density.

The convective heat-transfer coefficient is obtained from Newton's law of cooling,

$$q_w = h(T_w - T_\infty) \quad (8)$$

where

q_w = wall heat flux, h = convective heat-transfer coefficient, T_w = wall temperature, T_∞ = free-stream temperature.

The dimensionless heat-transfer parameter is represented by the Stanton number,

$$St = \frac{h}{(\rho C_p U_\infty)} \quad (9)$$

Where,

C_p denotes the specific heat capacity of the fluid.

For turbulent flow and $Pr \approx 1$, Reynolds proposed that the transport mechanisms of momentum and heat are analogous, leading to

$$St = \frac{C_f}{2} \quad (10)$$

Equation (10) is known as the classical Reynolds Analogy.

The Stanton number may also be expressed in terms of the Nusselt number,

$$St = \frac{Nu}{Re Pr} \quad (11)$$

Where,

$$Nu = \frac{hL}{k} \quad (12)$$

$$Re = \frac{\rho U_\infty L}{\mu} \quad (13)$$

$$Pr = \frac{C_p \mu}{k} \quad (14)$$

Substituting Equation (11) into Equation (10) gives

$$\frac{Nu}{RePr} = \frac{Cf}{2} \quad (15)$$

Therefore,

$$Nu = \frac{Cf}{2} RePr \quad (16)$$

Equation (16) establishes a direct relationship between heat transfer and momentum transfer and forms the basis of many engineering heat-transfer correlations.

3.2 Reynolds Analogy for Nanofluids

For nanofluids, the effective thermophysical properties differ from those of conventional fluids due to the presence of suspended nanoparticles. Consequently,

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_p \quad (17)$$

where

ϕ = nanoparticle volume fraction,

ρ_f = density of the base fluid,

ρ_p = density of nanoparticles.

Similarly, the effective thermal conductivity may be expressed as

$$k_{nf} > k_f$$

indicating that nanofluids generally possess higher thermal conductivity than the corresponding base fluids.

The Reynolds number for nanofluids becomes

$$Re_{nf} = \frac{\rho_{nf} U L}{\mu_{nf}} \quad (18)$$

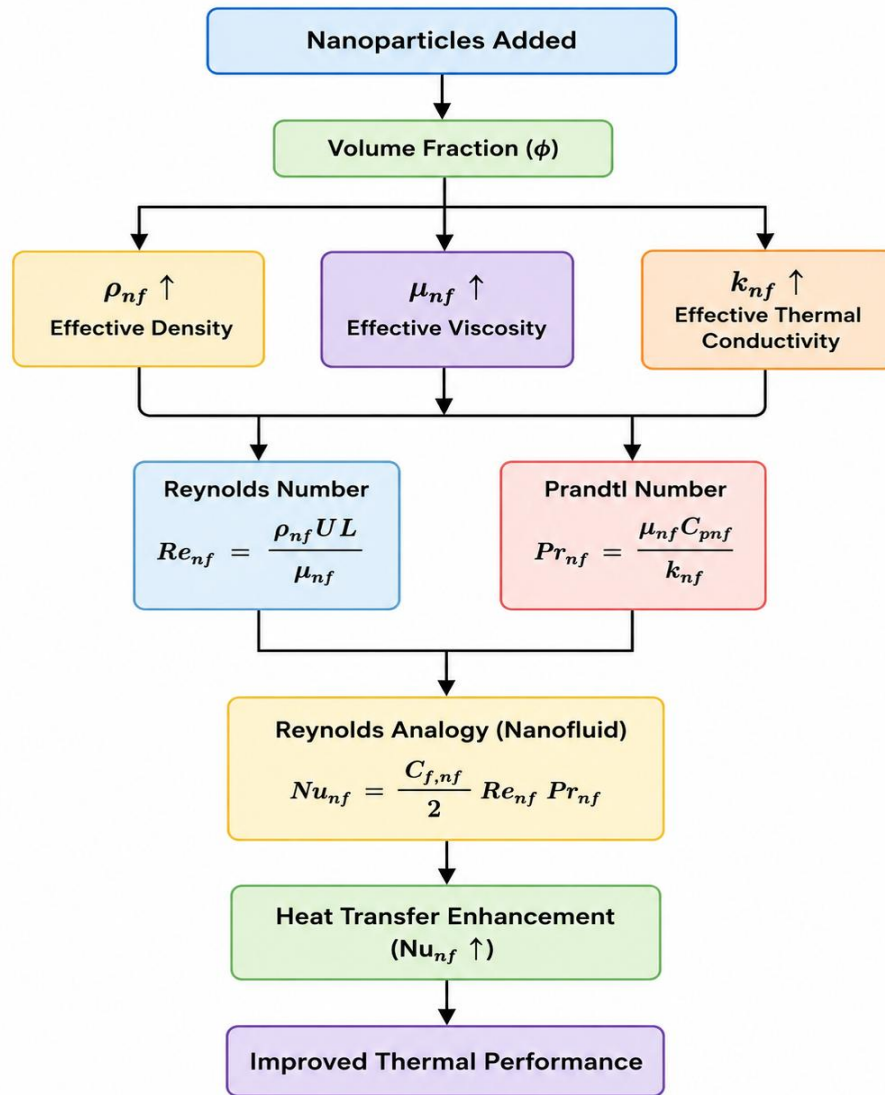
while the Nusselt number is

$$Nu_{nf} = \frac{h_{nf} L}{k_{nf}} \quad (19)$$

Substituting these modified properties into Equation (16) yields

$$Nu_{nf} = \frac{Cf,nf}{2} Re_{nf} Pr_{nf} \quad (20)$$

which provides a useful framework for estimating heat-transfer enhancement in nanofluid systems.



3.3 Engineering Significance

Reynolds analogy demonstrates that an increase in momentum transport generally leads to a corresponding increase in heat transfer. Consequently, factors that promote turbulence, such as higher Reynolds numbers, surface roughness, and nanoparticle dispersion, also enhance thermal transport. In nanofluid-filled vertical annuli subjected to thermal radiation, Reynolds analogy provides an important theoretical link between fluid flow behaviour and heat-transfer performance. Although the present study primarily focuses on natural convection, the analogy offers valuable insight into the interaction between hydrodynamic and thermal boundary layers and serves as a foundation for understanding more complex transport phenomena.

3.4 Application to Nanofluid Heat Transfer

For nanofluid flows, the Reynolds analogy remains useful because both momentum and thermal transport are influenced by nanoparticle dispersion within the base fluid. The presence of nanoparticles modifies the effective fluid properties, including density, viscosity, thermal conductivity, and specific heat capacity. Consequently, the Reynolds number, Prandtl number, and Nusselt number become functions of nanoparticle concentration.

The effective Reynolds number may be expressed as

$$Renf = \frac{\rho_{nf} U L}{\mu_{nf}} \quad (21)$$

while the corresponding Prandtl number becomes

$$Prnf = \frac{\mu_{nf} c_{p,nf}}{k_{nf}} \quad (22)$$

where the subscript nf denotes nanofluid properties.

Substituting these effective properties into the Reynolds analogy yields

$$Nunf = \frac{Cf}{2} Renf Prnf \quad (23)$$

which provides a useful framework for estimating heat-transfer enhancement in nanofluid systems.

3.5 Significance in Vertical Annulus Heat Transfer

In the present study, Reynolds analogy provides an important theoretical link between fluid-flow behaviour and thermal transport characteristics in a vertical annulus. Although the primary focus is on natural convection under thermal radiation effects, the analogy highlights the fundamental relationship between momentum transport and heat transfer. Furthermore, it provides a basis for interpreting how changes in nanofluid properties influence convective heat-transfer performance. The resulting dimensionless formulation serves as a foundation for the subsequent analysis of heat transfer mechanisms in nanofluid-filled annular geometries subjected to thermal radiation (Mahian et al., 2019; Sheikholeslami, 2018).

4. Heat Exchanger Effectiveness

Heat exchangers are among the most important thermal devices used in engineering systems for transferring thermal energy between two fluids maintained at different temperatures. Applications of heat exchangers include power generation plants, refrigeration systems, chemical processing equipment, automotive radiators, solar energy collectors, and thermal management systems. The primary objective of a heat exchanger is to maximize heat transfer while minimizing energy losses and operating costs. Consequently, the evaluation of heat exchanger performance is a fundamental aspect of thermal system design and analysis (Cengel & Ghajar, 2020; Shah & Sekulic, 2017).

One of the most widely used performance indicators for heat exchangers is the effectiveness, which measures the ratio of the actual heat transfer rate to the maximum possible heat transfer rate. Heat exchanger effectiveness provides a direct assessment of thermal performance independent of the exchanger size and operating conditions.

The actual heat transfer rate between the hot and cold fluids is expressed as

It will render as:

$$Q = m_h c_{p,h} (T_{h,i} - T_{h,o}) \quad (24)$$

where:

Q = heat transfer rate

m_h = mass flow rate of the hot fluid

$c_{p,h}$ = specific heat capacity of the hot fluid

$T_{h,i}$ = hot fluid inlet temperature

$T_{h,o}$ = hot fluid outlet temperature.

or equivalently,

$$Q = m_c c_{p,c} (T_{c,o} - T_{c,i}) \quad (25)$$

where:

Q = heat transfer rate

m_c = mass flow rate of the cold fluid

$c_{p,c}$ = specific heat capacity of the cold fluid

$T_{c,o}$ = cold-fluid outlet temperature

$T_{c,i}$ = cold-fluid inlet temperature

The thermal capacity rates of the fluids are defined as

$$C_h = m_h c_{p,h} \quad (26)$$

and

$$C_c = m_c c_{p,c} \quad (27)$$

The minimum and maximum heat capacity rates are given by

$$C_{\min} = \min(C_h, C_c) \quad (28)$$

and

$$C_{\max} = \max(C_h, C_c) \quad (29)$$

respectively.

The maximum possible heat transfer occurs when the fluid possessing the smaller heat capacity rate experiences the maximum allowable temperature change. Therefore,

$$Q_{\max} = C_{\min} (T_{h,i} - T_{c,i}) \quad (30)$$

The heat exchanger effectiveness is then defined as

$$\varepsilon = \frac{Q}{Q_{\max}} \quad (31)$$

Substituting Equations (25) and (27) into Equation (31) yields

$$\varepsilon = \frac{m_c c_{p,c} (T_{c,o} - T_{c,i})}{C_{\min} (T_{h,i} - T_{c,i})} \quad (32)$$

Similarly,

$$\varepsilon = \frac{m_h c_{p,h} (T_{h,i} - T_{h,o})}{C_{\min} (T_{h,i} - T_{c,i})} \quad (33)$$

Equations (32) and (33) provide practical means for determining heat exchanger effectiveness from measured temperature data.

4.1 Number of Transfer Units (NTU) Method

The effectiveness–NTU method is widely employed in heat exchanger analysis because it allows thermal performance to be evaluated without prior knowledge of outlet temperatures. The Number of Transfer Units (NTU) is defined as

$$NTU = \frac{UA}{C_{\min}} \quad (34)$$

where U denotes the overall heat transfer coefficient and A represents the total heat transfer surface area.

The heat capacity ratio is defined as

$$C_r = \frac{C_{\min}}{C_{\max}} \quad (35)$$

The effectiveness can therefore be expressed as a function of NTU and the heat capacity ratio:

$$\varepsilon = f(NTU, C_r) \quad (36)$$

For a parallel-flow heat exchanger,

$$\varepsilon = \frac{[1 - \exp \{-NTU (1 + C_r)\}]}{(1 + C_r)} \quad (37)$$

where:

ε = heat exchanger effectiveness

NTU = Number of Transfer Units

C_r = Heat capacity rate ratio

whereas for a counter-flow heat exchanger,

$$\varepsilon = \frac{[1 - \exp \{-NTU (1 + C_r)\}]}{[1 - C_r \exp \{-NTU (1 - C_r)\}]} \quad (38)$$

where:

ε = heat exchanger effectiveness

NTU = Number of Transfer Units

$$C_r = \frac{C_{\min}}{C_{\max}}.$$

Counter-flow configurations generally exhibit higher effectiveness because they maintain a larger temperature difference between the two fluids throughout the heat exchange process (Shah & Sekulic, 2017).

4.2 Heat Exchanger Effectiveness in Nanofluid Systems

Recent advances in nanotechnology have led to the development of nanofluids as advanced heat-transfer media. Nanofluids are produced by dispersing nanoparticles within conventional base fluids such as water, ethylene glycol, or oil. The presence of nanoparticles significantly enhances thermal conductivity and consequently improves heat transfer performance (Mahian et al., 2019).

For nanofluid heat exchangers, the effective heat capacity rate may be expressed as

$$C_{nf} = m_{nf} C_{p, nf}$$

where m_{nf} is the nanofluid mass flow rate and $c_{p, nf}$ is the effective specific heat capacity.

The corresponding effectiveness becomes

$$\varepsilon_{nf} = \frac{Q_{nf}}{Q_{\max, nf}}.$$

where $\dot{Q}_{nf}$ represents the actual heat transfer achieved by the nanofluid heat exchanger. Experimental investigations have demonstrated that increasing nanoparticle concentration generally enhances the overall heat transfer coefficient, thereby increasing NTU and improving effectiveness. However, excessive nanoparticle loading may increase viscosity and pumping power requirements, leading to reduced thermal efficiency. Therefore, an optimal nanoparticle concentration must be selected to achieve maximum thermal performance (Sundar et al., 2016; Mahian et al., 2019).

4.3 Relevance to Vertical Annulus Heat Transfer

In vertical annular systems, heat exchanger effectiveness provides an important measure of the capability of the annulus to transfer thermal energy between heated and cooled boundaries. The effectiveness concept is particularly useful when evaluating the influence of thermal radiation, nanofluid concentration, and flow conditions on overall thermal performance. Furthermore, effectiveness analysis provides a direct connection between theoretical heat-transfer models and practical engineering applications involving annular heat exchangers and thermal energy systems.

Figure 4 illustrates the relationship between heat transfer rate, thermal capacity rate, NTU, and effectiveness in a typical heat exchanger system.

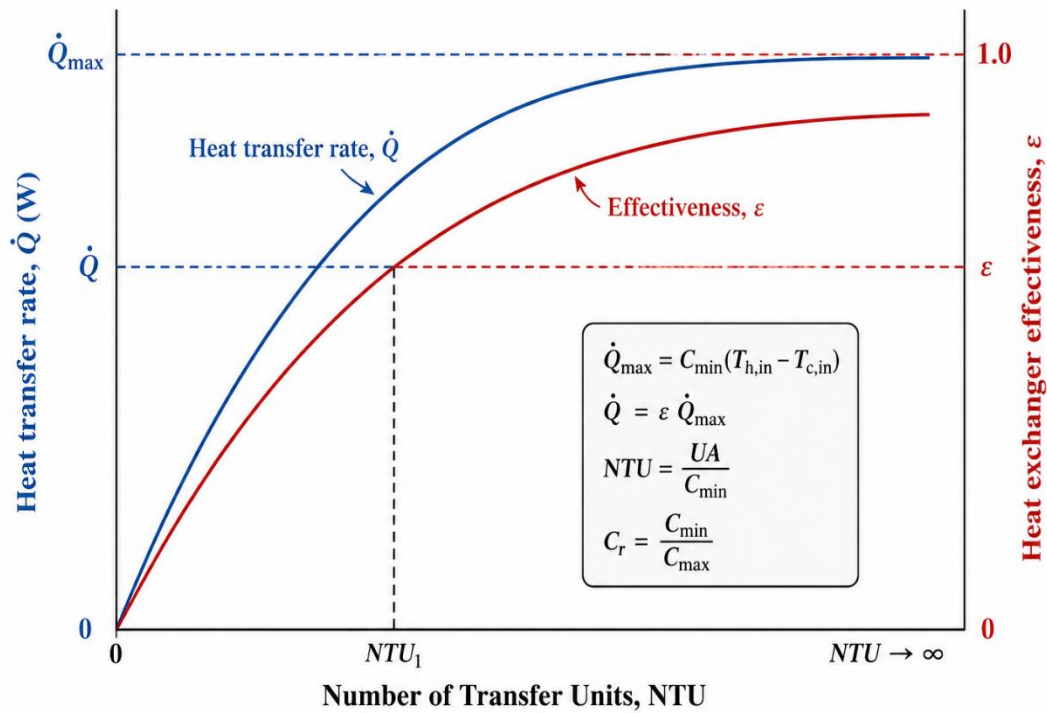


Figure 4. Schematic relationship between heat transfer rate, NTU, and heat exchanger effectiveness.

5. Conclusion

This study has presented a comprehensive mathematical analysis of convective heat transfer phenomena in nanofluid systems, with emphasis on forced convection, dimensional analysis, Reynolds analogy, and heat exchanger effectiveness. The investigation demonstrated that the use of dimensionless parameters such as the Reynolds number, Prandtl number, Nusselt number, Stanton number, and Number of Transfer Units (NTU) provides a systematic framework for characterizing and predicting thermal transport behaviour in engineering applications.

The dimensional analysis established generalized heat-transfer correlations that relate fluid properties, flow conditions, and geometric parameters to convective performance. Reynolds analogy was shown

to provide an important theoretical connection between momentum transfer and heat transfer, enabling the estimation of thermal transport characteristics from fluid-flow behaviour. The extension of this analogy to nanofluids highlighted the influence of nanoparticle concentration on effective density, viscosity, thermal conductivity, and heat-transfer enhancement.

Furthermore, the effectiveness-NTU method was examined as a practical tool for evaluating heat exchanger performance. The analysis demonstrated that improvements in nanofluid thermal properties can increase the overall heat-transfer coefficient, resulting in higher NTU values and enhanced heat exchanger effectiveness. However, excessive nanoparticle loading may increase viscosity and pumping power requirements, indicating the need for optimal nanoparticle concentrations.

Generally, the mathematical formulations developed in this study provide valuable insight into the mechanisms governing heat transfer in nanofluid-filled systems. The results contribute to a better understanding of thermal transport processes and offer a theoretical foundation for the design and optimization of advanced heat exchangers, annular thermal systems, and energy-efficient engineering applications involving nanofluids and thermal radiation effects. Future research may extend the present analysis to transient flows, hybrid nanofluids, variable thermophysical properties, and fractional-order heat-transfer models for more realistic thermal system predictions.

References

- Cengel, Y. A., & Ghajar, A. J. (2020). *Heat and Mass Transfer: Fundamentals and Applications* (6th ed.). McGraw-Hill Education.
- Hazeri-Mahmel, N., Shekari, Y., & Tayebi, A. (2020). Three-dimensional analysis of forced convection of Newtonian and non-Newtonian nanofluids through a horizontal pipe. *International Communications in Heat and Mass Transfer*, 121, 105119.
- Incropera, F. P., Bergman, T. L., Lavine, A. S., & DeWitt, D. P. (2017). *Fundamentals of Heat and Mass Transfer* (8th ed.). Wiley.
- Kakaç, S., Liu, H., & Pramuanjaroenkij, A. (2012). *Heat Exchangers: Selection, Rating and Thermal Design* (3rd ed.). CRC Press.
- Karwa, R. (2017). *Heat and Mass Transfer*. Springer.
- Mahian, O., Kolsi, L., Amani, M., Estellé, P., Ahmadi, G., Kleinstreuer, C., Marshall, J., Taylor, R. A., Niazmand, H., Wongwises, S., & Pop, I. (2019). Recent advances in modeling and simulation of nanofluid flows—Part II: Applications. *Physics Reports*, 791, 1–59.
- Mahian, O., Kolsi, L., Amani, M., et al. (2019). Recent advances in modeling and simulation of nanofluid heat transfer. *Applied Thermal Engineering*, 150, 104–123.
- Rashad, A. M. (2018). Natural convection and heat transfer enhancement using nanofluids. *Journal of Thermal Analysis and Calorimetry*, 132, 789–804.
- Shah, R. K., & Sekulic, D. P. (2017). *Fundamentals of Heat Exchanger Design*. Wiley.
- Sheikholeslami, M. (2018). Numerical investigation of nanofluid heat transfer enhancement. *Computer Methods in Applied Mechanics and Engineering*, 337, 82–95.
- Sundar, L. S., Sharma, K. V., & Singh, M. K. (2016). Experimental heat transfer studies of nanofluids. *Renewable and Sustainable Energy Reviews*, 68, 185–198.
- Sundar, L. S., Sharma, K. V., Singh, M. K., & Sousa, A. C. M. (2016). Hybrid nanofluids and their applications as advanced heat transfer fluids: A review. *Renewable and Sustainable Energy Reviews*, 68, 185–198.
- White, F. M. (2016). *Fluid Mechanics* (8th ed.). McGraw-Hill Education.
- Xuan, Y., Li, Q., & Yu, W. (2018). Nanofluid heat transfer enhancement mechanisms and applications. *International Journal of Heat and Mass Transfer*, 121, 437–451.