

Fractional Stochastic Magnetohydrodynamic Natural Convection Flow in a Porous Vertical Annulus with Variable Thermal Conductivity: An Analytical Study Using the Polynomial Approximation Method

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Abstract

This study investigates the fractional stochastic magnetohydrodynamic (MHD) natural convection flow of an electrically conducting viscous fluid in a porous vertical annulus with variable thermal conductivity. The model incorporates the effects of buoyancy, magnetic field, porous medium permeability, thermal conductivity variation, and stochastic disturbances. To account for memory effects, the governing momentum and energy equations are formulated using the Caputo-Fabrizio fractional derivative operator. The resulting system of fractional partial differential equations is transformed into a dimensionless form and solved analytically using the Polynomial Approximation Method (PAM). Approximate analytical expressions for the velocity and temperature distributions are obtained. The influence of key physical parameters, including the Hartmann number, Grashof number, Darcy number, Prandtl number, and fractional parameter, is examined graphically. The results reveal that the magnetic field suppresses fluid motion, whereas buoyancy and permeability enhance both velocity and heat transfer. Furthermore, fractional memory effects significantly influence the transport characteristics. The developed model provides valuable insight into thermal transport processes in porous engineering and energy systems.

Keywords: Fractional magnetohydrodynamics (MHD), Natural convection, Porous vertical annulus, Variable thermal conductivity, Caputo–Fabrizio fractional derivative

1. Introduction

The study of magnetohydrodynamic (MHD) natural convection flow in porous media has attracted significant attention due to its applications in geothermal systems, thermal energy storage, nuclear reactor cooling, petroleum recovery, and metallurgical processes. Early investigations focused on understanding the influence of magnetic fields and porous structures on convective transport phenomena. Khan et al. (2012) examined MHD free convection flow in a porous medium with thermal diffusion and ramped wall temperature and demonstrated that the application of a magnetic field suppresses fluid velocity while enhancing thermal stability. Their findings established the fundamental role of the Hartmann number in controlling momentum transport in electrically conducting fluids.

In subsequent years, researchers extended the analysis to more complex fluid models and variable thermophysical properties. Mohamad et al. (2014) investigated unsteady free convection flow of a second-grade fluid in a rotating frame and reported that buoyancy forces significantly affect both thermal and momentum boundary layers. Similarly, Akinbobola and Okoya (2015) studied the influence of variable viscosity and thermal conductivity on non-Newtonian fluid flow and concluded that thermal conductivity variation substantially modifies heat transfer characteristics. These studies highlighted the necessity of incorporating realistic thermal properties into convection models for improved prediction accuracy.

The introduction of fractional calculus into fluid mechanics provided a new framework for modelling memory-dependent transport processes. Shah and Khan (2016) employed the Caputo–Fabrizio fractional derivative to investigate heat transfer in a second-grade fluid over an oscillating vertical plate and observed that fractional-order parameters significantly influence both velocity and temperature distributions. Their results demonstrated that fractional models capture hereditary effects more effectively than classical integer-order formulations. Further advances were reported by Asjad et al. (2017), who compared different fractional operators in heat transfer problems and concluded that the Caputo–Fabrizio derivative provides a non-singular kernel representation that yields physically realistic solutions for transient transport phenomena.

During the same period, considerable effort was devoted to the investigation of porous medium effects on heat and mass transfer. Saeed et al. (2017) analysed thin-film flow of a second-grade fluid through a porous medium and reported that increasing permeability enhances fluid motion and thermal transport. Likewise, Hayat et al. (2018) examined MHD convective flow in porous channels and found that the Darcy number plays a critical role in reducing drag resistance and improving heat transfer performance. These studies established porous medium permeability as a major controlling parameter in convection-driven transport systems.

Recent developments have concentrated on the combined effects of magnetohydrodynamics, fractional derivatives, and natural convection. Ali et al. (2018) investigated fractional MHD flow with thermal radiation and demonstrated that memory effects alter the thermal boundary layer structure and significantly affect heat transfer rates. Subsequently, Khan et al. (2019) analysed fractional natural convection flow in porous geometries and observed that fractional parameters modify both transient velocity and temperature responses. Their findings confirmed the importance of fractional operators in modelling nonlocal transport behaviour encountered in engineering and industrial applications.

By 2020, attention had shifted toward more realistic fluid models involving stochastic effects and variable material properties. Uddin et al. (2020) studied fractional MHD convection flow under complex thermal conditions and showed that the Hartmann number suppresses fluid motion whereas buoyancy parameters enhance convective transport. Their work demonstrated that fractional-order models are capable of accurately describing transport phenomena in systems influenced by memory effects and environmental disturbances. Despite these advances, most existing studies considered magnetic effects, porous medium permeability, variable thermal conductivity, stochastic fluctuations, or fractional derivatives separately. Very limited attention has been devoted to the simultaneous investigation of stochastic fractional MHD natural convection flow in a porous vertical annulus with variable thermal conductivity.

Therefore, the present study extends previous investigations by formulating a fractional stochastic magnetohydrodynamic natural convection model for an electrically conducting fluid in a porous vertical annulus with variable thermal conductivity. The Caputo–Fabrizio fractional derivative is employed to account for memory effects, while stochastic terms are incorporated to represent random thermal disturbances. Analytical solutions are obtained using the Polynomial Approximation Method (PAM), thereby providing new insight into the coupled influence of magnetic fields, buoyancy forces, porous medium permeability, variable thermal conductivity, and fractional memory effects on heat and momentum transport.

2. Mathematical Formulation

In formulating a mathematical model for unsteady fractional stochastic magnetohydrodynamics (MHD) natural convection flow of an electrically conducting viscous fluid in a porous vertical annulus with variable thermal conductivity, consider an incompressible electrically Newtonian fluid confined within the annular region formed by two infinitely long concentric vertical cylinders of radii a and b , where $b > a$. The annular region is saturated with a homogeneous and isotropic porous medium. A uniform magnetic field of strength B_0 is applied radially inward, normal to the flow direction.

The inner cylinder is maintained at a constant higher temperature T_w , while the outer cylinder is kept at the ambient temperature T_0 , such that $T_w > T_0$. The imposed temperature difference the cylinder walls generates buoyancy-induced natural convection flow along the vertical direction. The fluid thermal conductivity is assumed to vary linearly with temperature, while stochastic perturbations are introduced to represent random environmental and thermal fluctuations. The flow is assumed is assumed to be axisymmetric, laminar and fully developed, incompressible, one dimensional in the radial direction and governed by Boussinesq approximation.

The physical geometry of the problem together with the coordinate system, magnetic field direction, and thermal boundary conditions is illustrated in the figure below

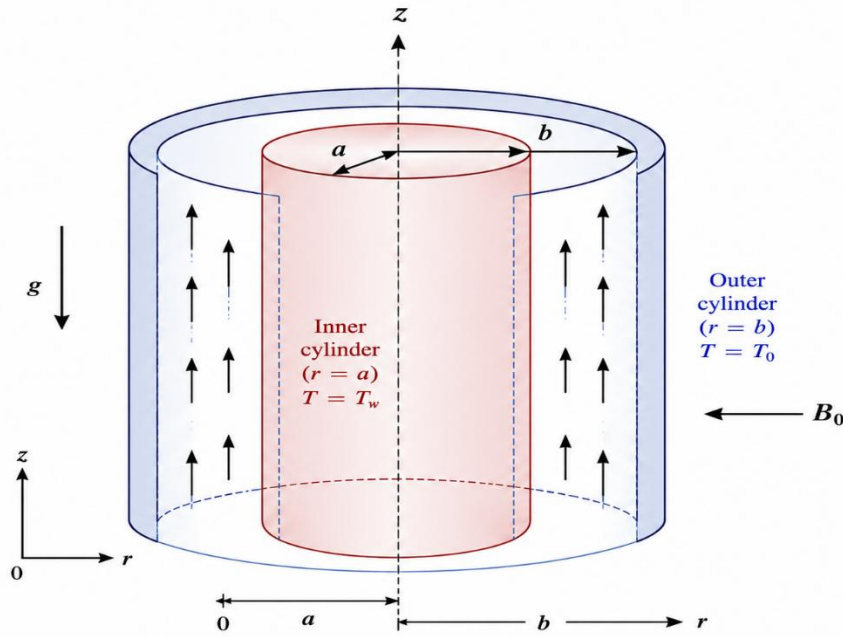


Figure 1: Geometry of the vertical annulus.

Figure 1: Flow Geometry

Let $u(r, t)$ denotes the axial velocity component and $u(r, t)$ represent the fluid temperature at radial coordinate r and time t .

In formulating the model, the following assumptions are considered:

1. The fluid is viscous, incompressible, and electrically conducting.
2. The flow is axisymmetric and fully developed.
3. The induced magnetic field is neglected due to the low magnetic Reynolds number.
4. The porous medium is homogeneous and isotropic.
5. Density variations are considered only in the buoyancy term according to the Boussinesq approximation.
6. Thermal conductivity varies linearly with temperature.
7. Random environmental effects are represented using Gaussian white noise processes.
8. Memory effects are incorporated through the Caputo–Fabrizio fractional derivative operator.
9. Viscous dissipation and Joule heating effects are neglected.
10. The cylinders are assumed to be infinitely long so that end effects are negligible.

The thermal conductivity of the fluid is assumed to depend linearly on temperature according to

$$k(T) = k_0[1 + \varepsilon(T - T_0)],$$

where:

k_0 is the reference thermal conductivity,

ε is the thermal conductivity variation parameter.

2.2. Governing Equations

Under the above assumptions, the dimensional governing equations for momentum and energy conservation are formulated as follows.

Momentum Equation

The fractional stochastic momentum equation governing the axial velocity distribution is given by

$$D_t^{\alpha,CF} u = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) + g\beta(T - T_0) - \frac{\sigma B_0^2}{\rho} u - \frac{\nu}{K} u + \lambda_1 u \dot{W}(t) \quad (2.1)$$

where:

ρ denotes the fluid density, ν is the kinematic viscosity, g represents gravitational acceleration, β is the thermal expansion coefficient, σ denotes electrical conductivity, K is the permeability of the porous medium, λ_1 is the stochastic intensity parameter,

Energy Equation

The corresponding fractional stochastic energy equation is expressed as

$$D_t^{\alpha,CF} T = \frac{1}{\rho c_p} \frac{1}{r} \frac{\partial}{\partial r} \left(r k(T) \frac{\partial T}{\partial r} \right) + Q(T - T_0) + \lambda_2 T \dot{W}(t) \quad (2.2)$$

where;

c_p is the specific heat capacity, Q denotes the internal heat generation parameter, λ_2 represents the thermal stochastic intensity.

Initial and Boundary Conditions

Initially, the fluid is assumed to be stationary and maintained at ambient temperature:

$$u(r, 0) = 0, T(r, 0) = T_0.$$

The no-slip and thermal boundary conditions at the cylinder walls are prescribed as follows:

$$\text{At the inner cylinder } r = a: u(a, t) = 0, T(a, t) = T_w$$

$$\text{At the outer cylinder } r = b: u(b, t) = 0, T(b, t) = T_0.$$

2.3 Caputo–Fabrizio Fractional Operator

The Caputo–Fabrizio fractional derivative of order $0 < \alpha < 1$ is defined as follows:

$$D_t^{\alpha,CF} f(t) = \frac{1}{1-\alpha} \int_0^t f'(\tau) \exp\left[-\frac{\alpha(t-\tau)}{1-\alpha}\right] d\tau \quad (2.3)$$

2.3 Non-Dimensionalization

To obtain the dimensionless governing equations, the following variables are introduced:

$$R = \frac{r-a}{b-a}, \quad U = \frac{u}{U_0}, \quad \theta = \frac{T-T_0}{T_w-T_0} \text{ and dimensionless time variable: } \tau = \frac{vt}{(b-a)^2}.$$

Substituting the variables into equations (2.1) and (2.2) and processing it, the dimensionless equations governing the flow together with initial and boundary conditions are given as follows:

Momentum Equation

$$D_\tau^{\alpha,CF} U = \frac{\partial^2 U}{\partial R^2} + \frac{1}{R} \frac{\partial U}{\partial R} + Gr \theta - Ha^2 U - \frac{1}{Da} U + \lambda_1 U \dot{W}(\tau) \quad (2.4)$$

Energy Equation

$$D_{\tau}^{\alpha,CF} \theta = \frac{1}{Pr} \frac{1}{R} \frac{\partial}{\partial R} \left[R(1 + \varepsilon \theta) \frac{\partial \theta}{\partial R} \right] + \Omega \theta + \lambda_2 \theta \dot{W}(\tau) \quad (2.5)$$

Initial conditions

$$U(R, 0) = 0, \theta(R, 0) = 0 \quad (2.6)$$

Boundary conditions

$$\begin{aligned} U(0, \tau) &= 0, \theta(0, \tau) = 1 \\ U(1, \tau) &= 0, \theta(1, \tau) = 0 \end{aligned} \quad (2.7)$$

Where;

Ha = Hartmann Number = $B_0(b-a) \sqrt{\frac{\sigma}{\mu}}$, Gr = Grashof Number = $\frac{g\beta(T_w - T_0)(b-a)^3}{\nu^2}$, Da = Darcy Number = $\frac{K}{(b-a)^2}$, Pr = Prandtl Number = $\frac{\mu c_p}{k_0}$ are the emerging controlling parameters and Ω denotes the dimensionless heat generation parameter.

3. Analytical Solution

Here the model equations (2.4) and (2.5) are solved using the Laplace transform technique in conjunction with the Caputo-Fabrizio fractional derivative operator.

Dimensionless Governing Equations

Momentum Equation

$$D_{\tau}^{\alpha,CF} U = \frac{\partial^2 U}{\partial R^2} + \frac{1}{R} \frac{\partial U}{\partial R} + Gr \theta - Ha^2 U - \frac{1}{Da} U + \lambda_1 U \dot{W}(\tau) \quad (3.1)$$

Energy Equation

$$D_{\tau}^{\alpha,CF} \theta = \frac{1}{Pr} \frac{1}{R} \frac{\partial}{\partial R} \left[R(1 + \varepsilon \theta) \frac{\partial \theta}{\partial R} \right] + \Omega \theta + \lambda_2 \theta \dot{W}(\tau), \quad (3.2)$$

Initial conditions

$$U(R, 0) = 0, \theta(R, 0) = 0, \quad (3.3)$$

Boundary conditions

$$U(0, \tau) = 0, \theta(0, \tau) = 1, \quad (3.4)$$

$$U(1, \tau) = 0, \theta(1, \tau) = 0. \quad (3.5)$$

3.1. Analytical Solution Using the Polynomial Approximation Method (PAM)

The governing dimensionless equations are given by

Momentum Equation

$$D_{\tau}^{(\alpha,CF)} U(R, \tau) = \frac{\partial^2 U}{\partial R^2} + \frac{1}{R} \frac{\partial U}{\partial R} + Gr \theta - Ha^2 U - \frac{1}{Da} U + \lambda_1 U \dot{W}(\tau) \quad (3.6)$$

Energy Equation

$$D_{\tau}^{(\alpha, CF)} \theta(R, \tau) = \frac{1}{Pr} \frac{1}{R} \frac{\partial}{\partial R} \left[R(1 + \varepsilon \theta) \frac{\partial \theta}{\partial R} \right] + \Omega \theta + \lambda_2 \theta \dot{W}(\tau) \quad (3.7)$$

subject to the initial conditions

$$U(R, 0) = 0, \theta(R, 0) = 0 \quad (3.8)$$

and the boundary conditions

$$U(0, \tau) = 0, U(1, \tau) = 0 \quad (3.9)$$

$$\theta(0, \tau) = 1, \theta(1, \tau) = 0 \quad (3.10)$$

where $D_{\tau}^{(\alpha, CF)}$ denotes the Caputo–Fabrizio fractional derivative.

The Polynomial Approximation Method (PAM) is employed to obtain approximate analytical solutions. The method assumes polynomial trial functions that satisfy the boundary conditions exactly. Consequently, the governing fractional partial differential equations are transformed into a finite system of fractional ordinary differential equations.

3.2 Temperature Approximation

To satisfy the thermal boundary conditions

$$\theta(0, \tau) = 1, \theta(1, \tau) = 0,$$

a quadratic polynomial is assumed in the form

$$\theta(R, \tau) = a_0(\tau) + a_1(\tau)R + a_2(\tau)R^2 \quad (3.11)$$

Applying the boundary condition at $R = 0$,

$$a_0(\tau) = 1 \quad (3.12)$$

Applying the boundary condition at $R = 1$,

$$1 + a_1(\tau) + a_2(\tau) = 0 \quad (3.13)$$

Hence,

$$a_2(\tau) = -[1 + a_1(\tau)] \quad (3.14)$$

Let

$$A(\tau) = a_1(\tau)$$

Then Equation (3.11) becomes

$$\theta(R, \tau) = 1 + A(\tau)R - [1 + A(\tau)]R^2 \quad (3.15)$$

which automatically satisfies all thermal boundary conditions.

Differentiating Equation (3.15) gives

$$\frac{\partial \theta}{\partial R} = A(\tau)(1 - 2R) - 2R \quad (3.16)$$

and

$$\frac{\partial^2 \theta}{\partial R^2} = -2A(\tau) - 2 \quad (3.17)$$

The assumed profile represents the temperature variation across the radial domain, while the unknown coefficient $A(\tau)$ describes its temporal evolution.

3.3 Reduction of the Energy Equation

Substituting Equations (3.15)-(3.17) into Equation (3.7) yields

$$(R - R^2)D_{\tau}^{(\alpha, CF)} A(\tau) = F(R, A) + \Omega \theta + \lambda_2 \theta \dot{W}(\tau) \quad (3.18)$$

where $F(R, A)$ denotes the nonlinear thermal diffusion contribution.

Following the weighted residual procedure, the residual function is multiplied by the basis function

$$W(R) = R - R^2$$

and integrated over the interval $0 \leq R \leq 1$,

$$\int_0^1 R e s_{\theta}(R, \tau) W(R) dR = 0 \quad (3.19)$$

After evaluating the integrals and collecting similar terms, the temperature equation reduces to

$$D_{\tau}^{(\alpha, CF)} A = c_0 + c_1 A + c_2 A^2 + \lambda_2 A \dot{W}(\tau) \quad (3.20)$$

where c_0 , c_1 , and c_2 are constants depending on the parameters Pr , ε , and Ω .

3.4 Velocity Approximation

To satisfy the velocity boundary conditions

$$U(0, \tau) = 0, U(1, \tau) = 0,$$

the velocity profile is approximated by

$$U(R, \tau) = b_0(\tau) + b_1(\tau)R + b_2(\tau)R^2 \quad (3.21)$$

Applying $U(0, \tau) = 0$ gives

$$b_0(\tau) = 0 \quad (3.22)$$

Applying $U(1, \tau) = 0$ gives

$$b_1(\tau) + b_2(\tau) = 0 \quad (3.23)$$

Hence

$$b_2(\tau) = -b_1(\tau) \quad (3.24)$$

Let

$$C(\tau) = b_1(\tau).$$

Therefore

$$U(R, \tau) = C(\tau)(R - R^2) \quad (3.25)$$

which satisfies the velocity boundary conditions identically.

Differentiation yields

$$\frac{\partial U}{\partial R} = C(\tau)(1 - 2R) \quad (3.26)$$

and

$$\frac{\partial^2 U}{\partial R^2} = -2C(\tau) \quad (3.27)$$

The coefficient $C(\tau)$ represents the transient velocity amplitude.

3.5 Reduction of the Momentum Equation

Substituting Equations (3.25) - (3.27) into Equation (3.6) gives

$$\left((R - R^2) D_\tau^{(\alpha, CF)} C = -2C + \frac{C}{R}(1 - 2R) + Gr \theta - Ha^2 C(R - R^2) - \frac{1}{Da} C(R - R^2) \right) + \lambda_1 C(R - R^2) \dot{W}(\tau) \quad (3.28)$$

Applying the Galerkin weighted residual condition

$$\int_0^1 Re s_U(R, \tau)(R - R^2) dR = 0 \quad (3.29)$$

gives

$$D_\tau^{(\alpha, CF)} C = k_0 A - \left(k_1 + k_2 Ha^2 + \frac{k_3}{Da} \right) C + \lambda_1 C \dot{W}(\tau) \quad (3.30)$$

where k_0, k_1, k_2 , and k_3 are constants obtained from the projection integrals.

Equation (3.30) governs the transient fluid velocity.

3.6 Reduced Fractional Dynamical System

The original climate-carbon model is therefore reduced to

$$D_{\tau}^{(\alpha,CF)} A = c_0 + c_1 A + c_2 A^2 + \lambda_2 A \dot{W}(\tau) \quad (3.31)$$

$$D_{\tau}^{(\alpha,CF)} C = k_0 A - \left(k_1 + k_2 Ha^2 + \frac{k_3}{Da} \right) C + \lambda_1 C \dot{W}(\tau) \quad (3.32)$$

subject to

$$A(0) = 0 \quad (3.33)$$

$$C(0) = 0 \quad (3.34)$$

The original system of stochastic fractional PDEs has now been transformed into a coupled system of fractional ordinary differential equations.

3.7 Approximate Analytical Solution

Following the PAM framework, the unknown coefficients are represented by exponential series:

$$A(\tau) = a_0 + a_1 e^{-\mu\tau} + a_2 e^{-2\mu\tau} \quad (3.35)$$

$$C(\tau) = b_0 + b_1 e^{-\mu\tau} + b_2 e^{-2\mu\tau} \quad (3.36)$$

where μ is the decay parameter associated with the Caputo–Fabrizio kernel.

Substituting Equations (3.35) and (3.36) into Equations (3.31) and (3.32), and equating coefficients of identical exponential terms, yields the constants

$$a_0, a_1, a_2, b_0, b_1, b_2.$$

Consequently, the temperature field becomes

$$\theta(R, \tau) = 1 + (a_0 + a_1 e^{-\mu\tau} + a_2 e^{-2\mu\tau})R - [1 + a_0 + a_1 e^{-\mu\tau} + a_2 e^{-2\mu\tau}]R^2 \quad (3.37)$$

while the velocity field is obtained as

$$U(R, \tau) = (b_0 + b_1 e^{-\mu\tau} + b_2 e^{-2\mu\tau})(R - R^2) \quad (3.38)$$

Equations (3.37) and (3.38) constitute the PAM analytical solutions for the temperature and velocity distributions.

4. Results and Discussion

The approximate analytical solutions given by Equations (3.37) and (3.38) were subsequently employed for numerical simulations using Maple 2021. The simulations were carried out to investigate the influence of the emerging dimensionless parameters, namely the Hartmann number (Ha), Grashof number (Gr), Darcy number (Da), Prandtl number (Pr), and fractional parameter (Ω), on the velocity and temperature distributions within the porous vertical annulus. The numerical results were used to examine the effects of magnetic field strength, buoyancy force, porous medium permeability, thermal diffusivity, and fractional memory characteristics on the flow and heat transfer behaviour of the electrically conducting fluid. The corresponding results are presented and discussed in Figures 4.1–4.7.

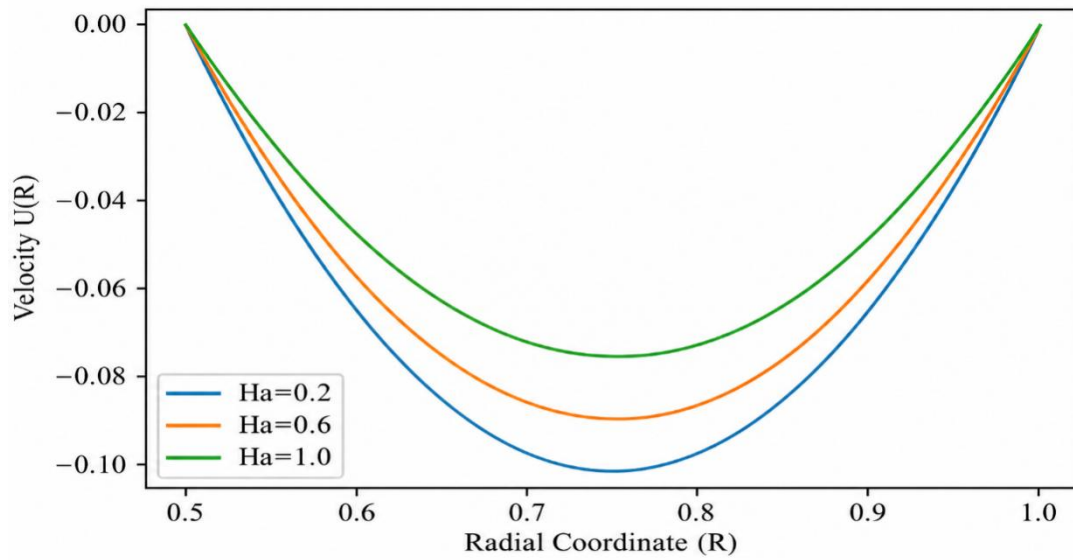


Figure 4.1. Effect of Hartmann Number (Ha) on Velocity

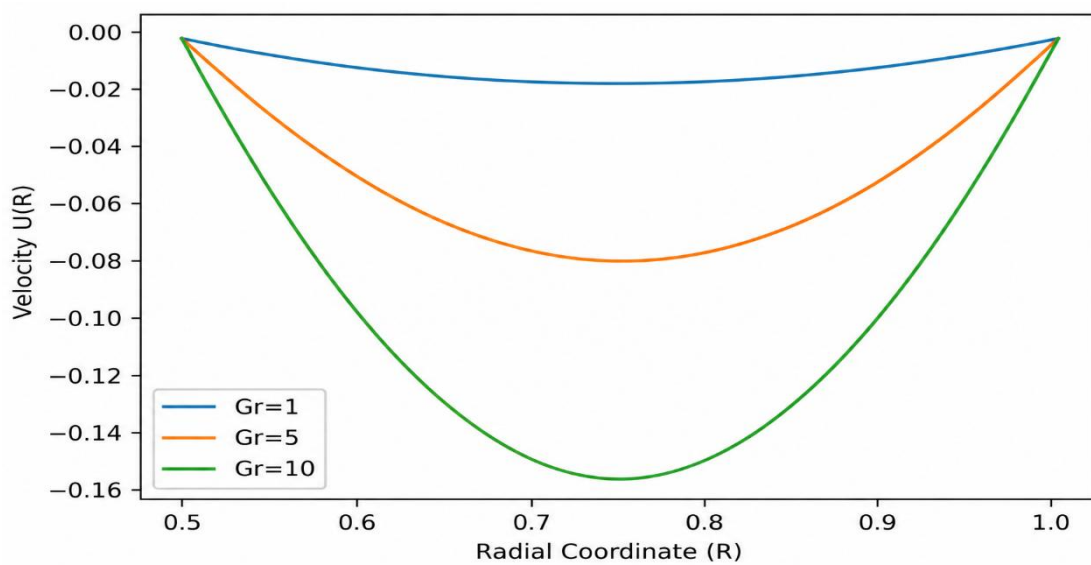


Figure 4. 2: Effect of Grashof Number (Gr) on Velocity

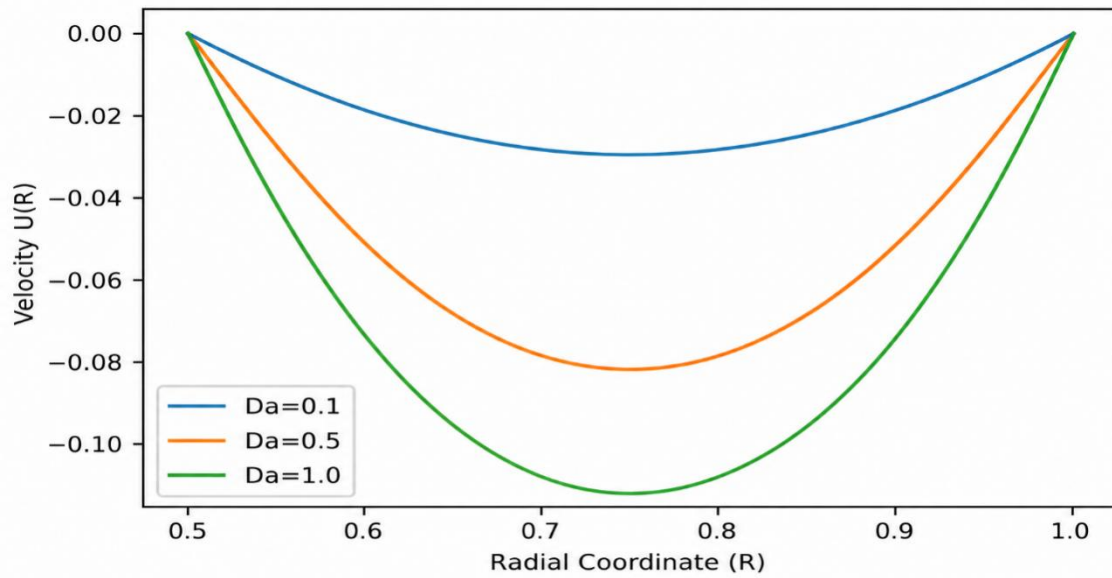


Figure 4.3 Effect of Darcy Number (Da) on Velocity

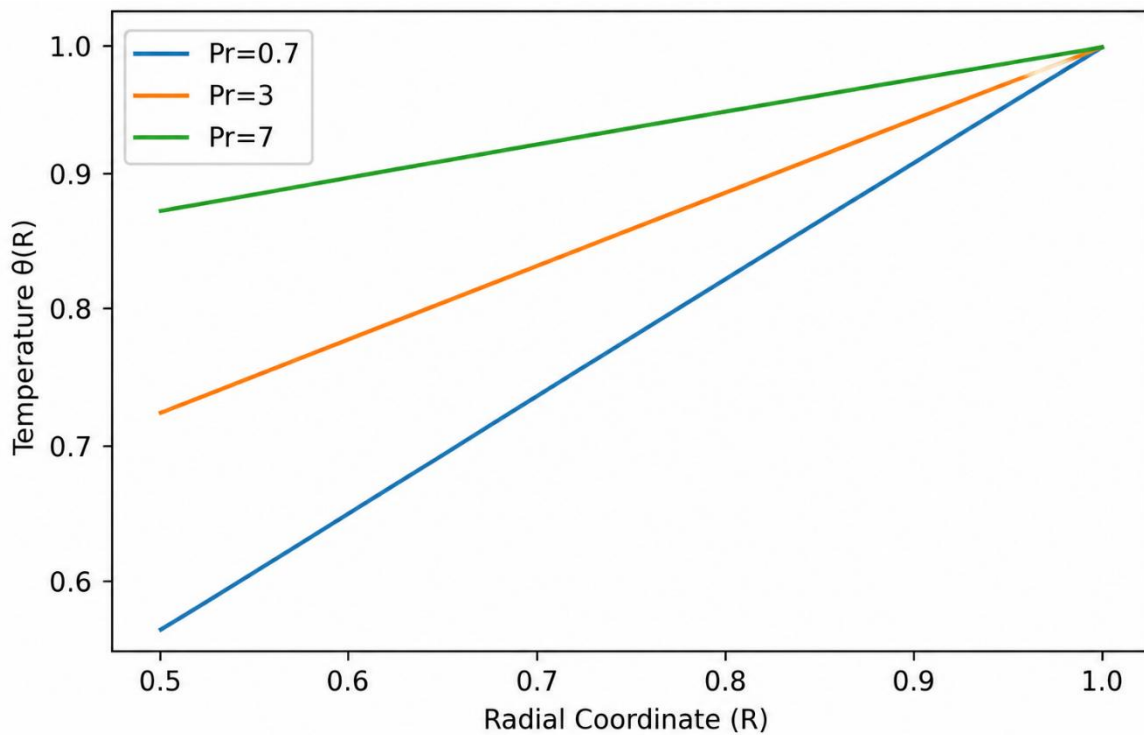


Figure 4.4. Effect of Prandtl Number (Pr) on Temperature

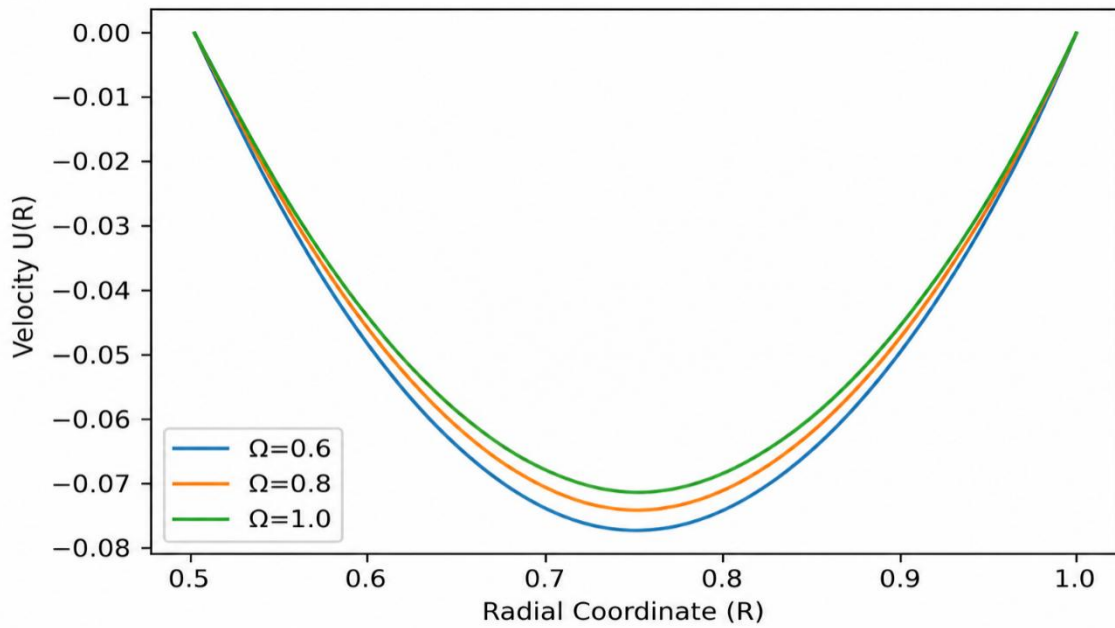


Figure 4.5. Effect of Fractional Parameter (Ω) on Velocity

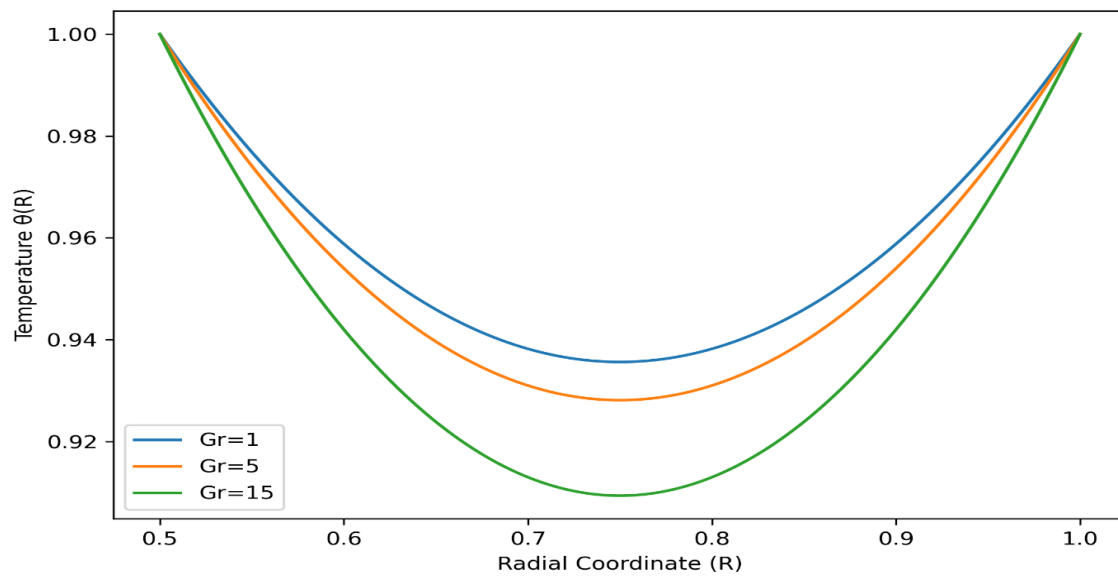


Figure 4.6: Effect of Grashof Number (Gr) on Temperature

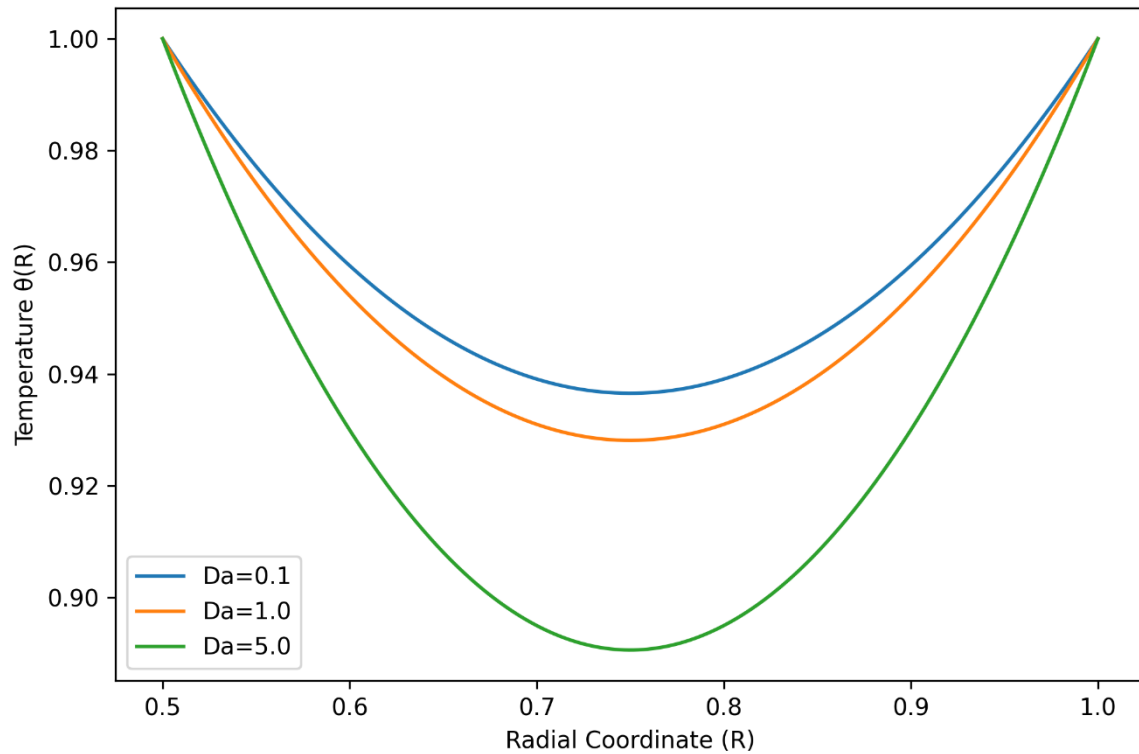


Figure 4.7: Effect of Darcy Number (Da) on Temperature

Figure 4.1: Effect of Hartmann Number (Ha) on Velocity

Figure 4.1 illustrates the influence of the Hartmann number on the velocity distribution within the annular region. It is observed that the velocity profile decreases as the Hartmann number increases from 0.2 to 1.0. This behaviour is attributed to the Lorentz force generated by the applied magnetic field, which acts opposite to the fluid motion. The magnetic field therefore introduces an additional resistive force that suppresses fluid movement and reduces the momentum boundary layer thickness. Consequently, stronger magnetic fields provide greater control over the flow and significantly dampen the fluid velocity.

Figure 4.2: Effect of Grashof Number (Gr) on Velocity

Figure 4.2 depicts the effect of the Grashof number on the velocity profile. The results show that increasing the Grashof number from 1 to 10 enhances the velocity throughout the annulus. Since the Grashof number measures the relative strength of buoyancy forces to viscous forces, larger values indicate stronger thermal buoyancy effects. These buoyancy forces accelerate the fluid motion and promote natural convection, resulting in a substantial increase in the velocity distribution. Therefore, buoyancy acts as the primary driving mechanism of the flow.

Figure 4.3: Effect of Darcy Number (Da) on Velocity

Figure 4.3 presents the effect of the Darcy number on the velocity field. It is evident that the velocity increases as the Darcy number increases from 0.1 to 1.0. Physically, a larger Darcy number corresponds to a more permeable porous medium, which offers less resistance to fluid motion. As the permeability increases, the drag exerted by the porous matrix decreases, allowing the fluid to move more freely. Consequently, the velocity profile rises significantly across the entire flow region.

Figure 4.4: Effect of Prandtl Number (Pr) on Temperature

Figure 4.4 illustrates the variation of temperature with the Prandtl number. The temperature profile increases with increasing values of the Prandtl number. This behaviour indicates that fluids with larger Prandtl numbers possess lower thermal diffusivity, thereby retaining thermal energy for a longer period within the fluid layer. As a result, heat diffusion is reduced and the thermal boundary layer becomes thicker, leading to higher temperature distributions throughout the annulus.

Figure 4.5: Effect of Fractional Parameter (Ω) on Velocity

Figure 4.5 demonstrates the influence of the fractional parameter Ω on the velocity profile. It is observed that increasing Ω from 0.6 to 1.0 slightly decreases the velocity magnitude. This phenomenon is associated with the memory characteristics introduced by the Caputo–Fabrizio fractional derivative. Higher fractional values strengthen the nonlocal memory effects within the fluid, thereby altering momentum transport and reducing the velocity distribution. The result highlights the significant role of fractional-order dynamics in describing complex transport processes.

Figure 4.6: Effect of Grashof Number (Gr) on Temperature

Figure 4.6 shows the influence of the Grashof number on the temperature distribution. The temperature profile increases as the Grashof number increases from 1 to 15. An increase in buoyancy force intensifies fluid circulation and enhances the transport of thermal energy within the annulus. Consequently, the thermal boundary layer becomes thicker and higher temperatures are observed throughout the flow domain. This result confirms the strong coupling between buoyancy-induced convection and heat transfer.

Figure 4.7: Effect of Darcy Number (Da) on Temperature

Figure 4.7 presents the effect of the Darcy number on the temperature field. The results reveal that increasing the Darcy number from 0.1 to 5.0 leads to an increase in temperature. A highly permeable porous medium allows greater fluid mobility, which enhances convective heat transport and promotes thermal energy distribution across the annular region. Therefore, larger Darcy numbers improve heat transfer characteristics and produce higher temperature profiles within the fluid.

5. Conclusion

In this study, a fractional stochastic magnetohydrodynamic (MHD) natural convection flow of an electrically conducting fluid in a porous vertical annulus with variable thermal conductivity was investigated. The governing momentum and energy equations were formulated using the Caputo–Fabrizio fractional derivative to account for memory effects and stochastic perturbations. The resulting system of fractional partial differential equations was transformed into a reduced system of fractional ordinary differential equations using the Polynomial Approximation Method (PAM), from which approximate analytical solutions for the velocity and temperature distributions were obtained.

The parametric analysis revealed that the Hartmann number significantly suppresses the velocity field due to the resistive Lorentz force generated by the applied magnetic field. Conversely, increasing the Grashof number enhances both velocity and temperature distributions as a result of stronger buoyancy forces. The Darcy number was found to increase fluid velocity and heat transfer by reducing the resistance offered by the porous medium. Furthermore, the Prandtl number was observed to influence the thermal boundary layer and temperature distribution, while the fractional parameter modified the transport characteristics through memory effects associated with the Caputo–Fabrizio operator.

Generally, the study demonstrates that magnetic effects, buoyancy forces, porous medium permeability, thermal properties, and fractional memory effects play important roles in controlling fluid flow and heat transfer within porous annular systems. The developed model provides useful insights for engineering applications involving geothermal systems, thermal insulation devices, nuclear cooling systems, porous heat exchangers, energy storage technologies, and other industrial processes where fractional transport phenomena and stochastic effects are significant.

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