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## Heat and Fluid Flow Analysis in a Porous Annular System Under Mixed Convection Flow

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### Abstract

*This study presents an analytical investigation of transient mixed convection flow of an incompressible viscous fluid in a vertical concentric annulus filled with porous materials. A mathematical model governing the flow and heat transfer processes was formulated based on the momentum and energy equations under several assumptions, Boussinesq approximation and fully developed laminar flow conditions. The governing partial differential equations corresponding to momentum and energy equations were non-dimensionalized using relevant dimensionless parameters such as the Grashof number, Reynolds number, Peclet number, and magnetic field parameter. The existence and uniqueness of the solution to the model were established, ensuring the validity of the mathematical formulation. Analytical solutions were then obtained using suitable transformation techniques. The influence of key physical parameters on velocity and temperature distributions was examined through graphical analysis. The results reveal that an increase in the magnetic field parameter reduces fluid velocity, while higher Grashof and Reynolds numbers enhance the flow velocity. Conversely, an increase in the Peclet number leads to a decrease in velocity but an increase in temperature distribution within the annulus. This study contributes to the understanding of transport phenomena in porous annular systems and provides useful insights for optimizing engineering processes involving heat and mass transfer in porous media.*

**Keywords:** Mixed Convection Flow, Porous Annulus, Magnetic Field Effect, Heat Transfer, Parameters

## 1.0 INTRODUCTION

Recent research studies on mixed convection flow in porous annuli have used both numerical and analytical methods. Hussain and Saihood (2015) conducted a numerical and experimental study on mixed convection heat transfer in a porous annulus with a rotating inner cylinder, demonstrating that rotation enhances heat transmission and alters flow patterns. Their findings emphasized the importance of boundary conditions and geometric configurations in controlling flow behaviour.

Similarly, Salman Ahmed et al. (2016) employed finite element analysis to study mixed convection and flow in a vertical porous annulus. Their studies demonstrated that the Peclet number, radiation parameter, and radius ratio all had a substantial impact on the annulus' thermal behaviour and flow structure. This study looked more into the role of thermal boundary conditions in porous systems. Moderres et al. (2017) extended the research by investigating double-diffusive mixed convection in a horizontal annulus partially filled with porous material. Their computer findings revealed that thermal and concentration gradients have a significant impact on convection patterns and heat transfer rates, highlighting the intricacies of transport processes in porous media.

Jha et al. (2018) found that the Darcy number and viscosity ratio are crucial in determining flow reversal and temperature distribution in non-Darcian mixed convection flow in a porous annulus. This paper is still relevant in confirming analytical approaches to convection problems. Additionally, Aibo Jha et al. (2018) proposed accurate solutions for non-Darcian mixed convection flow in an annulus partially filled with porous material. Their studies demonstrated that Darcy number and viscosity ratio are important factors in controlling flow reversal and temperature distribution. This study is still significant for confirming analytical methods to convection problems. Additionally, Aiboudi et al. (2015) investigated similarity solutions for mixed convection boundary layer flows in porous media. Their findings demonstrated the existence and uniqueness of solutions under specified conditions, highlighting the importance of mathematical modeling techniques in resolving difficult fluid flow problems.

Liu and He (2016) created a lattice Boltzmann model to describe convection heat transfer in porous media. Their findings showed that modern numerical methods can accurately reflect complex flow behaviour in porous systems, particularly in multidimensional topologies where analytical solutions are difficult to get. Experimental studies have also greatly aided our understanding of mixed convection in porous annuli. Hussein (2016) studied heat transfer qualities in a concentric annulus filled with a metallic porous media and discovered that differences in Reynolds number and heating conditions had a substantial impact on Nusselt number and temperature distribution.

More recently Rundora, L., *et al* (2018) examined the behaviour of transient convection flow in porous media and found that the combination of buoyancy forces and applied pressure gradients has a substantial impact on velocity and temperature profiles. The study also emphasized the role of dimensionless parameters like Grashof and Reynolds numbers in regulating flow characteristics. This study lends credence to the use of analytical tools in studying convection events in porous annular systems.

Recent studies have highlighted the significance of porous annular systems in engineering applications such as geothermal systems, heat exchangers, thermal insulation devices, nuclear reactors, and energy storage systems. The interaction of forced and free convection in porous media has a substantial impact on heat transfer efficiency and fluid transport properties. Mahdy and Chamkha (2014) argue that when heat radiation and viscous dissipation factors

are included in the governing equations, mixed convection flow in porous annuli becomes more complex. Their findings demonstrated that buoyant forces have a significant impact on velocity distribution and temperature gradients within porous channels. Likewise, Rashidi et al. (2013) analyzed heat transfer enhancement in porous annular geometries and reported that permeability and porosity parameters greatly influence the thermal efficiency of the system. These studies confirm the practical relevance of porous annular configurations in industrial and environmental heat transfer processes.

Several researchers have also investigated the impact of magnetic fields and transient effects on convection flow in porous annuli. Sheikholeslami et al. (2015) studied magnetohydrodynamic mixed convection flow in porous cavities and discovered that magnetic factors inhibit fluid velocity while increasing thermal boundary layer thickness. Their findings suggested that magnetic control may be used in engineering systems requiring regulated heat transfer. Similarly, Makinde and Eegunjobi (2011) investigated transient mixed convection flow in porous channels and concluded that increasing the Grashof number improves fluid velocity and heat transfer rates by enhancing buoyancy effects. In another similar work, Srinivasacharya and Kaladhar (2012) examined heat and mass transfer in porous media under convective boundary conditions and established that Peclet number variations significantly alter thermal diffusion characteristics. These contributions provide a strong theoretical foundation for the present study and support the need for further analytical investigation into porous annular flow systems.

Despite these advancements, the majority of existing studies rely primarily on numerical simulations, with few providing closed-form analytical solutions to transient mixed convection issues in porous annuli. Furthermore, little consideration has been given to the combined effects of magnetic field, Reynolds number, Peclet number, and Grashof number in a unified analytical framework. This gap inspires the current research, which aims to build an analytical model to assess the combined impacts.

## **2.0 Materials and Method**

### **2.1 Physical Model**

We consider a transient, laminar, and fully developed mixed convection flow of an incompressible viscous fluid in a vertical concentric annulus filled with a porous medium of constant porosity. The annulus is bounded by two coaxial cylinders of radii  $a$  (inner cylinder) and  $b$  (outer cylinder), such that  $a \leq r \leq b$ , where  $r$  denotes the radial coordinate.

The flow is considered to occur in the axial  $z$ -direction, propelled by a continuous pressure gradient. Thermal effects are introduced by temperature changes between the inner and outside walls, resulting in buoyant forces. Furthermore, an external magnetic field with uniform intensity  $B_0$  is introduced, altering the flow via electromagnetic damping. The fluid parameters are assumed constant, with the exception of density fluctuations in the buoyancy term, which are handled using the Boussinesq approximation.

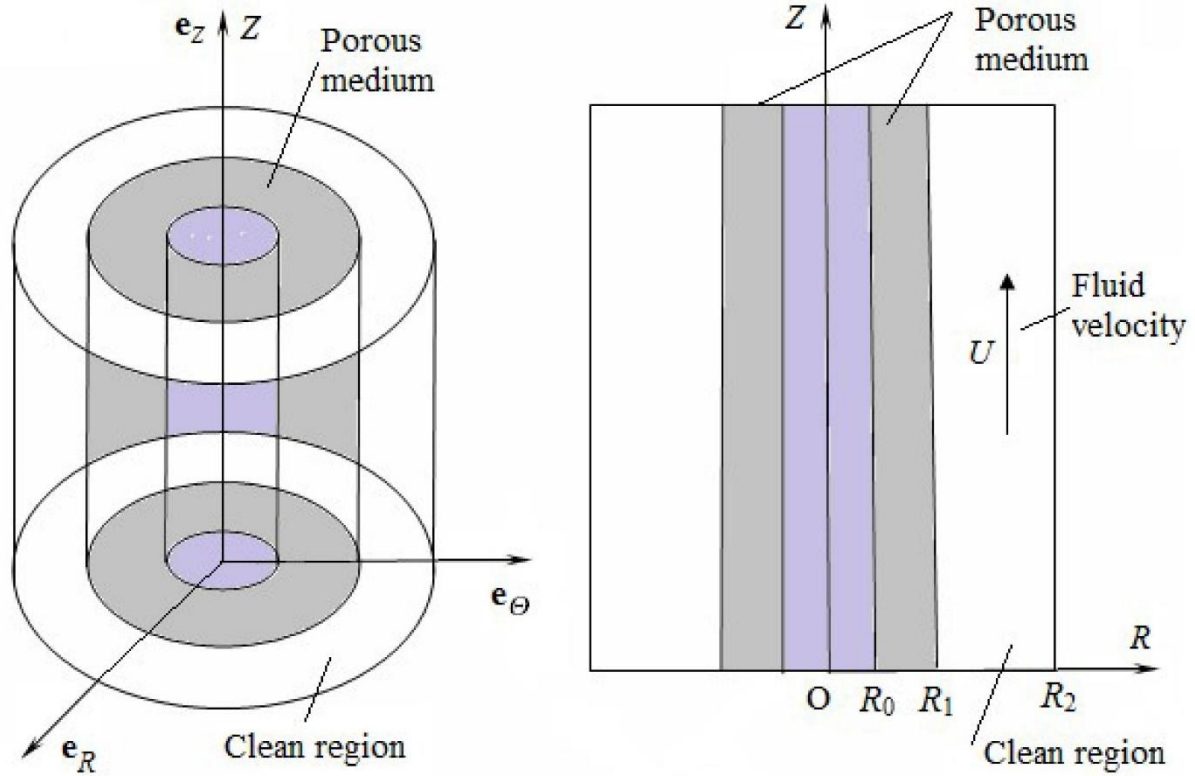


Figure 1: Flow Geometry

Under the above assumptions, the governing equations for momentum and energy are expressed as follows:

Momentum Equation

$$\frac{\partial u}{\partial t} = \nu \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) - \frac{1}{\rho} \frac{\partial p}{\partial z} + g\beta(T - T_0) - \frac{\sigma B_0^2}{\rho} u \quad (3.1)$$

Energy Equation

$$\frac{\partial T}{\partial t} = Pe \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) - u \frac{\partial T}{\partial z} \quad (3.2)$$

The initial and boundary conditions are as follows

$$\begin{aligned} u(r, 0) &= 0, T(r, 0) = T_0. \\ u(a, t) &= 0, u(b, t) = 0. \\ T(a, t) &= T_1, T(b, t) = T_2. \end{aligned}$$

where  $T_1$  and  $T_2$  represent the temperatures of the inner and outer cylinders, respectively. The variables and physical parameters used in the model are defined as follows:

$u(r, t)$  = axial velocity of the fluid,  $T(r, t)$  = temperature of the fluid,  $r$  = radial coordinate,  $z$  = axial coordinate,  $t$  = time,  $\nu$  = kinematic viscosity,  $\alpha$  = thermal diffusivity,  $\rho$  = fluid density,  $p$  = pressure,  $g$  = acceleration due to gravity,  $\beta$  = coefficient of thermal expansion,  $T_0$  = reference temperature,  $\sigma$  = electrical conductivity of the fluid,  $B_0$  = magnetic field strength

### 3.2 Non – Dimensionalization

To simplify the governing equations and identify the key physical parameters controlling the flow and heat transfer, the following dimensionless variables are introduced:

$$u' = \frac{u}{U}, \quad r' = \frac{r}{R}, \quad Z' = \frac{z}{L}, \quad \theta = \frac{T - T_o}{T_1 - T_o}, \quad t' = \frac{Ut}{L}$$

where  $R$  and  $L$  are the characteristic length Scales in the radial and axial directions, respectively, and  $U$  is a characteristic velocity.

Substituting these variables into the governing equations (3.1) and (3.2), and dropping the prime ('), we obtain the following dimensionless equations:

Momentum Equation

$$\frac{\partial u}{\partial t} = \frac{1}{Re} \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) - \frac{\partial p}{\partial z} + Gr\theta - Mu \quad (3.3)$$

Energy Equation

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pe} \left( \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} \right) - u \frac{\partial \theta}{\partial z} \quad (3.4)$$

The corresponding dimensionless boundary conditions become:

$$u(a, t) = 0, u(b, t) = 0. \\ \theta(a, t) = 1, \theta(b, t) = 0.$$

The dimensionless parameter appearing in the models are as follows:

$Re$  = Reynolds number,  $Gr$  = Grashof number,  $Pe$  = Peclet number,  $M$  = Magnetic parameter

### 2.3 Analytical Solution

The governing dimensionless momentum and energy equations are solved analytically using a regular perturbation method with an artificially introduced small parameter  $\epsilon$ , following the approach of Mohammed et al. (2015).

Here, we approximated (3.3) and (3.4) by embedded an artificial parameter  $\epsilon$  as follows (Mohammed et al., 2015)

$$\frac{\partial \theta}{\partial t} = Pe \left( \frac{\partial^2 \theta}{\partial r^2} + \frac{\epsilon}{r} \frac{\partial \theta}{\partial r} \right) \quad (3.5)$$

$$\frac{\partial u}{\partial t} = k_1 + \frac{1}{Re} \left( \frac{\partial^2 u}{\partial r^2} + \frac{\epsilon}{r} \frac{\partial u}{\partial r} \right) + Gr\theta - \frac{M}{Re} u \quad (3.6)$$

Let  $0 < \epsilon < 1, B = b \in \text{such that}$

$$\left. \begin{aligned} u(r, t) &= u_0(r, t) + \epsilon u_1(r, t) + \dots \\ \theta(r, t) &= \theta_0(r, t) + \epsilon \theta_1(r, t) + \dots \end{aligned} \right\} \quad (3.7)$$

Using (3.7) in (3.6) and (3.5) and collecting the like power of  $\epsilon$ , we have

$$\epsilon^0$$

$$\frac{\partial \theta_0}{\partial t} = Pe \frac{\partial^2 \theta_0}{\partial r^2} \quad (3.8)$$

$$\theta_0(r, 0) = r(1-r), \quad A \frac{\partial \theta_0}{\partial r} \Big|_{r=a} = \beta, \quad \theta_0(1, t) = 1$$

$$\frac{\partial u_0}{\partial t} = k_1 + \frac{1}{Re} \frac{\partial^2 u_0}{\partial r^2} + Gr \theta_0 - \frac{M}{Re} u_0 \quad (3.9)$$

$$u_0(r, 0) = r(1-r), \quad u_0(a, t) = 0, \quad u_0(1, t) = \sigma$$

$$\in^1$$

$$\frac{\partial \theta_1}{\partial t} = Pe \frac{\partial^2 \theta_1}{\partial r^2} + \frac{\alpha}{r} \frac{\partial \theta_0}{\partial r} \quad (3.10)$$

$$\theta_1(r, 0) = 0, \quad A \frac{\partial \theta_1}{\partial r} \Big|_{r=a} + b \theta_0(r, t) \Big|_{r=a} = 0, \quad \theta_1(1, t) = 0$$

$$\frac{\partial u_1}{\partial t} = \frac{1}{Re} \frac{\partial^2 u_1}{\partial r^2} + \frac{1}{Re} \frac{\partial u_0}{\partial t} + Gr \theta_1 - \frac{M}{Re} u_1 \quad (3.11)$$

$$u_1(r, 0) = 0, \quad u_1(a, t) = 0, \quad u_1(1, t) = 0$$

To simplify the computational domain, the transformation

$$x = r - a.$$

is introduced, which maps the interval  $r \in [a, 1]$  to  $x \in [0, L]$ , where  $L = 1 - a$ .

Under this transformation, the energy equation becomes:

$$\frac{\partial \theta_0}{\partial t} = Pe \frac{\partial^2 \theta_0}{\partial x^2} \quad (3.12)$$

Solving equation (3.8) – (3.12). We obtain the final solution in (3.14) and (3.15) as

$$U(r, t) = \sum_{n=1}^{\infty} U_n(t) \sin n\pi x + \frac{\sigma \alpha}{1-a} + \sum_{n=1}^{\infty} (P_1 + P_2 + P_3 + P_4) \quad (3.13)$$

$$\theta(r, t) = \frac{1}{3}(1+a-2a^2) + \sum_{n=1}^{\infty} a_n e^{-Pe} \left( \frac{(2n-1)\pi}{2(1-a)} \right)^2 \cos \frac{(2n-1)\pi}{2(1-a)}(r-a) +$$

$$\beta(r-1) + a +$$

$$\frac{2}{1-a} \left\{ (Pe \beta \ln a) \tau + \right.$$

$$\left. \frac{1}{2} Pe (2n-1) \pi^2 (1-a)^2 e^{-Pe (2n-1) \pi^2 (1-a)^2 t} - \frac{1}{2} n=1 \infty a_n Pe (2n-1) \pi^2 (1-a)^2 \cdot \frac{1}{2} \right.$$

$$\left. - a Pe \beta \ln a \tau + \frac{1}{2} Pe (2n-1) \pi^2 (1-a)^2 e^{-Pe (2n-1) \pi^2 (1-a)^2 t} - \frac{1}{2} n=1 \infty a_n Pe (2n-1) \pi^2 \right.$$

$$\left. (1-a)^2 \cdot \right\} \quad (3.14)$$

Where;

$$P_1 = \eta \left( \frac{e^{At}}{A^2} - \frac{\tau}{A} - \frac{1}{A^2} \right) \int_0^t T_1 T_2 (e^{\beta \tau} e^{A(t-\tau)} - e^{A(t-\tau)}) d\tau.$$

$$P_2 = T_1 T_2 \left[ \frac{1}{B-A} [e^{Bt} - e^{At}] + \frac{1}{A} [1 - e^{At}] \int_0^t \Phi (e^{A(t-\tau)} - e^{A(t-\tau)} e^{B\tau}) d\tau. \right.$$

$$\left. P_3 = \Phi \left[ \frac{1}{B-A} [e^{At} - e^{Bt}] + \frac{1}{A} [e^{At} - 1] \int_0^t \omega \tau e^{B\tau} e^{A(t-\tau)} d\tau. \right. \right.$$

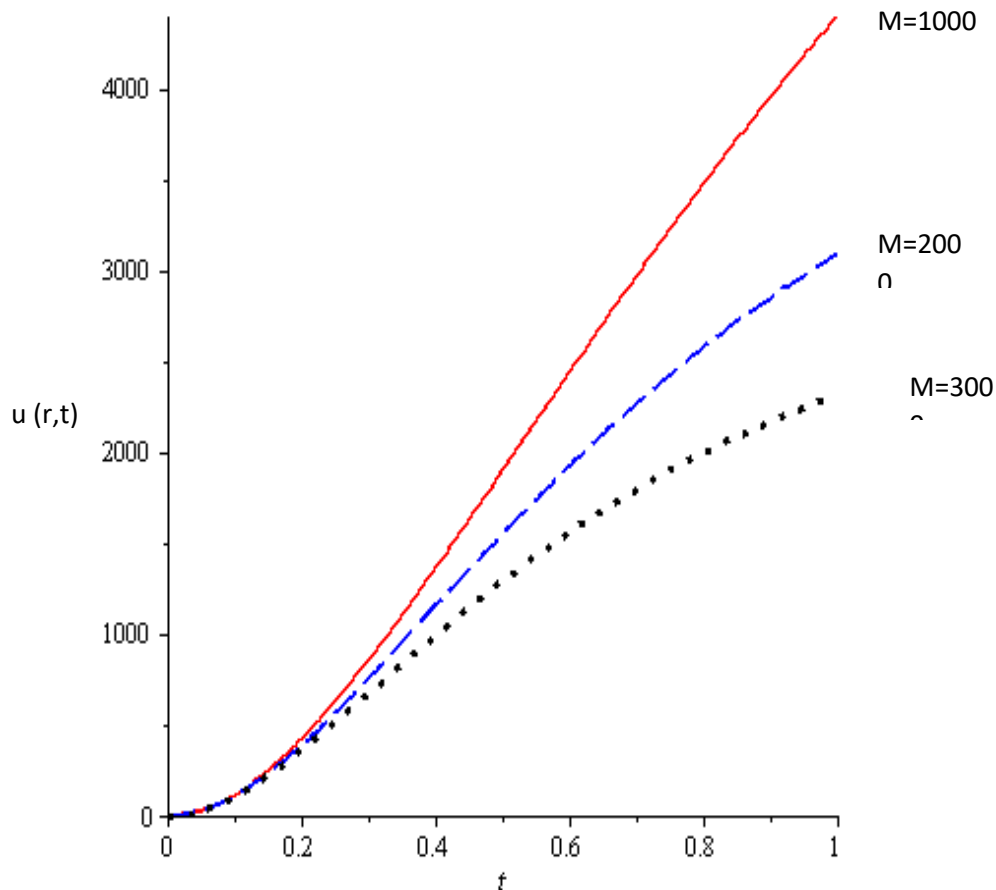
$$P_4 = \omega \left[ \frac{te^{Bt}}{B-A} - \frac{e^{Bt}}{(B-A)^2} + \frac{e^{At}}{(B-A)^2} \right].$$

The computations were done on equation (3.13) –(3.14) using computer symbolic algebraic package Maple 2017 version.

### 3.0 Result and Discussion

The analytical solutions for velocity and temperature distributions were used to investigate the effect of key dimensionless parameters, namely the magnetic parameter (M), Grashof number (Gr), Reynolds number (Re), and Peclet number (Pe), on flow behaviour within the porous annular system. Simulation was performed on equation (3.13) – (3.14) using computer symbolic algebraic package Maple 2017 version and the results were graphically presented and interpreted in accordance with the scientific principles that control mixed convection flow.

Figure 4.1 shows the effect of Magnetic field parameter M on velocity



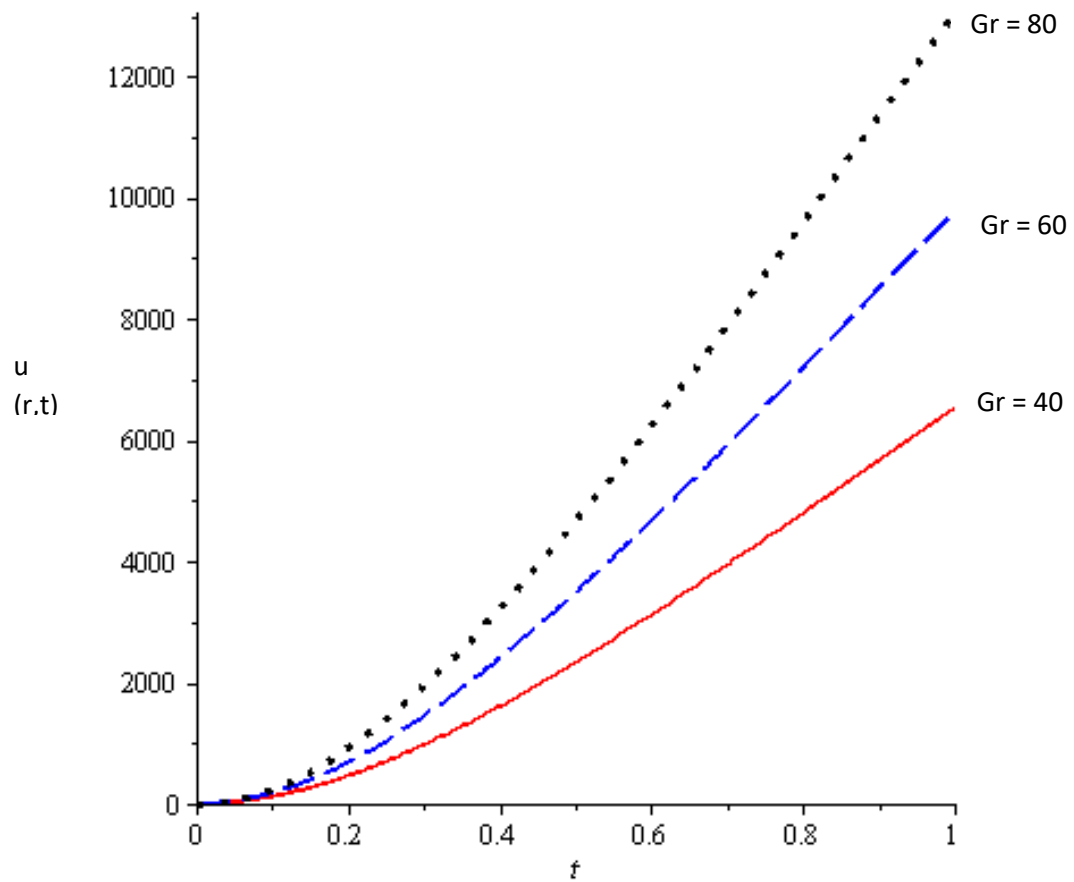


Figure 4.2 shows the effect of Grashof  $Gr$  on velocity.

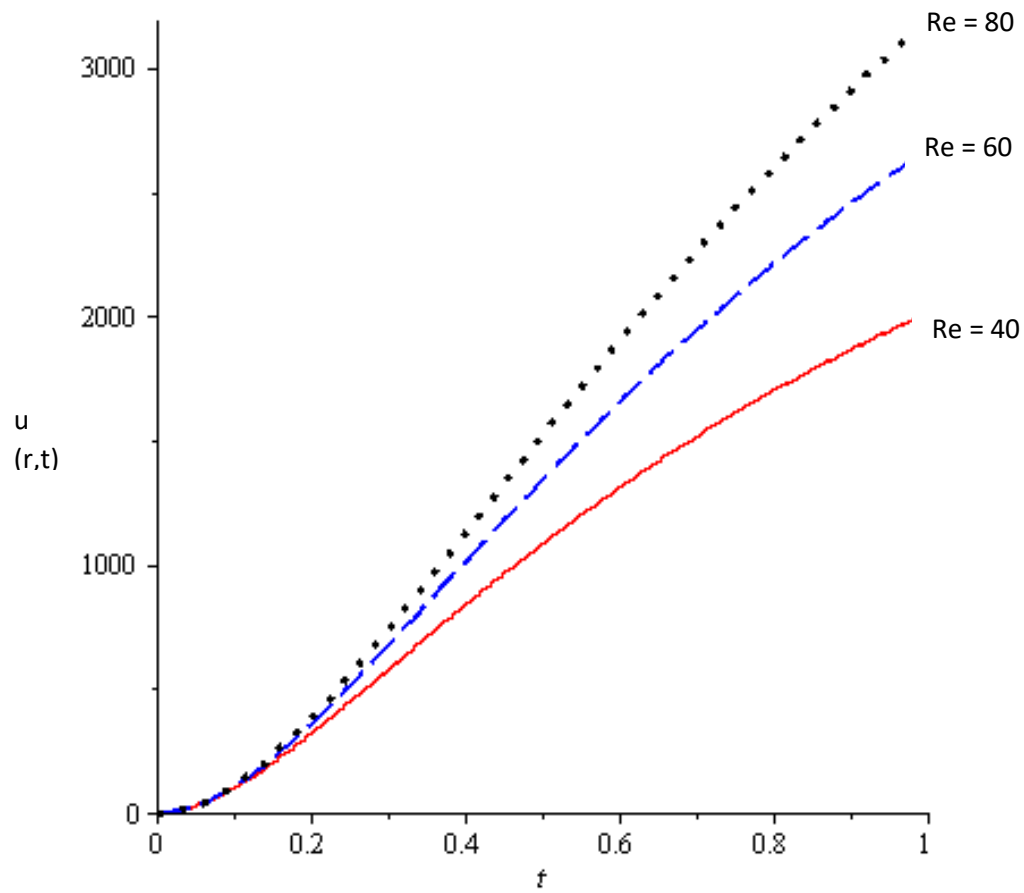


Figure 4.3 shows the effect of Reynolds number  $Re$  on velocity.

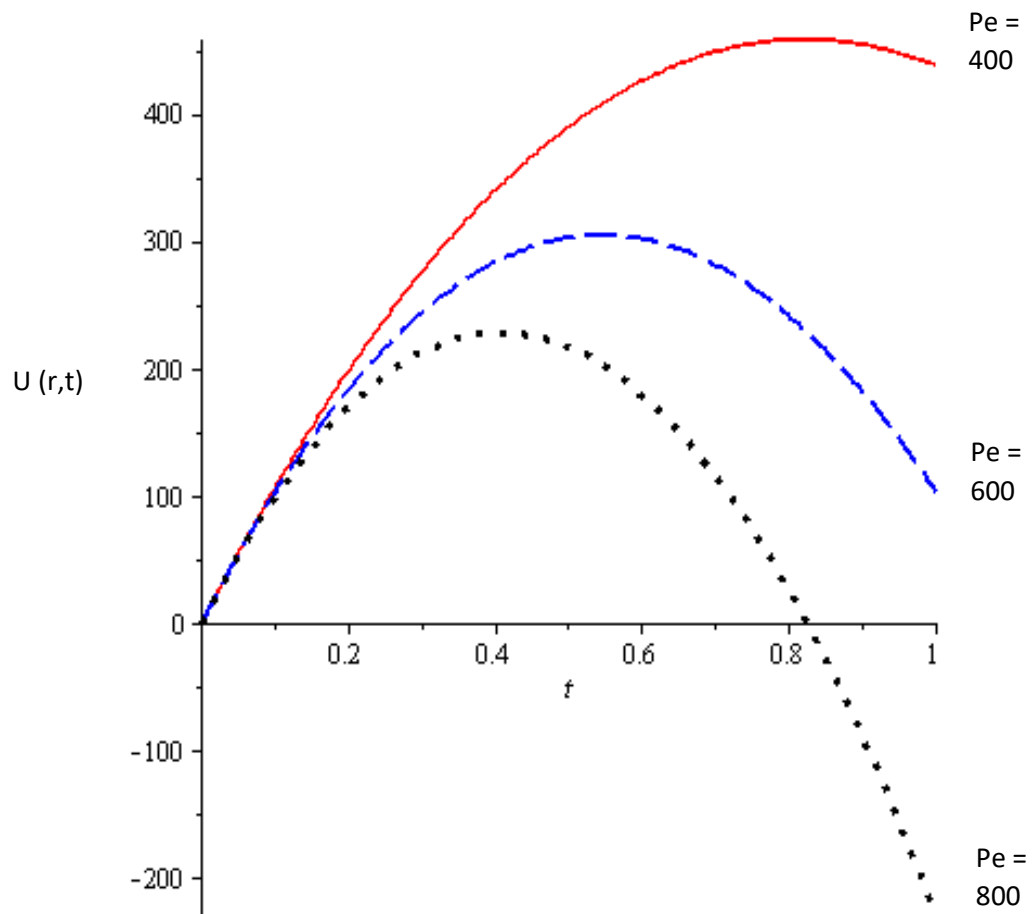


Figure 4.4 shows the effect of Peclet number  $Pe$  on velocity.

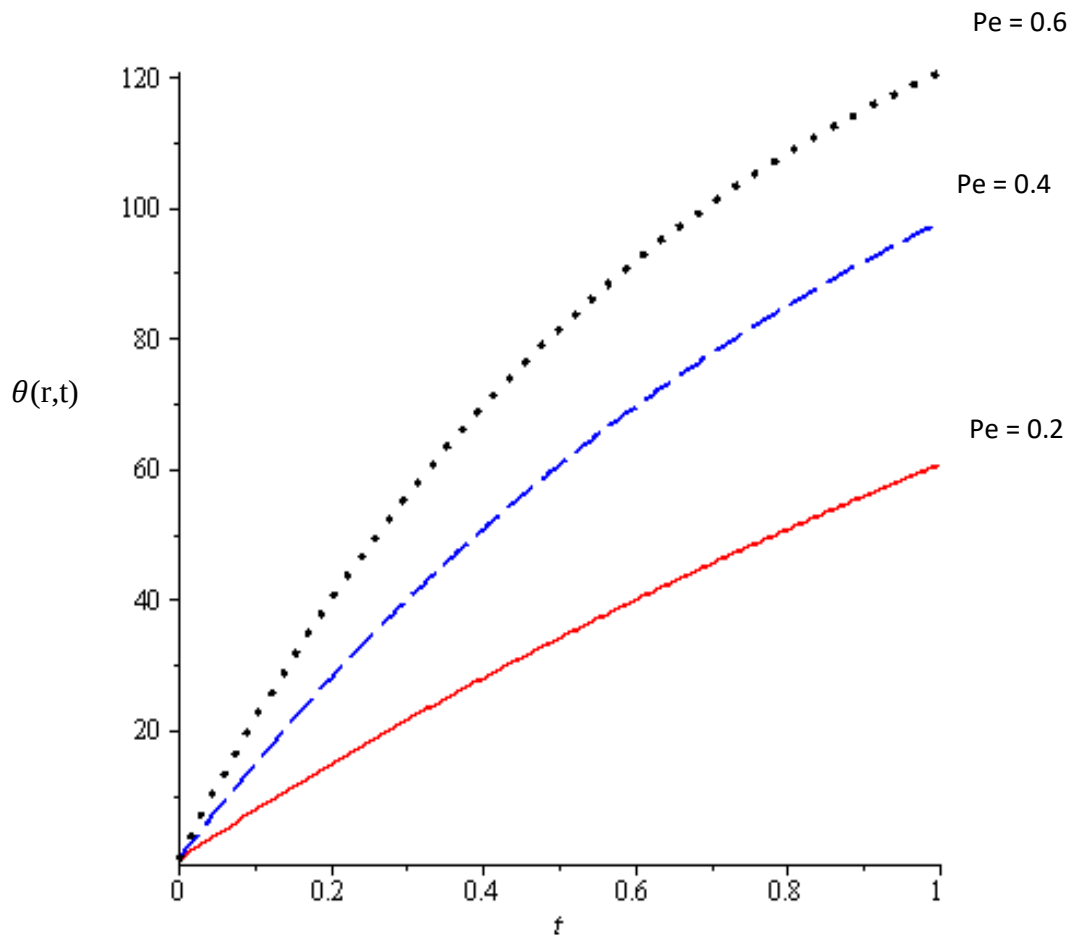


Figure 4.5 shows the effect of Peclet number  $Pe$  on temperature.

Figure 4.1 depicts the variation in velocity profile with increasing magnetic parameter ( $M$ ). Fluid velocity is observed to drop dramatically as  $M$  increases. This behaviour is attributed to the presence of the Lorentz force induced by the applied magnetic field, which acts as a resistive force opposing the fluid motion. As a result, increasing magnetic field strength improves flow retardation, which dampens velocity. This finding demonstrates that magnetic fields can be utilized to successfully control fluid flow in electrically conducting fluids within porous media.

Figure 4.2 depicts the effect of the Grashof number ( $Gr$ ) on the velocity distribution. An increase in  $Gr$  causes a substantial rise in fluid velocity. Physically, the Grashof number represents the buoyancy-to-viscous force ratio. Higher  $Gr$  values indicate stronger buoyant forces caused by temperature differences, promoting upward fluid motion and intensifying convective flow within the annulus.

Figure 4.3 depicts how Reynolds number ( $Re$ ) affects velocity. The results reveal that velocity increases as  $Re$  increases. Because the Reynolds number measures the ratio of inertial forces to viscous forces, larger numbers indicate greater flow momentum. This causes a rise in fluid velocity, implying that forced convection dominates the flow regime at higher Reynolds numbers.

Figure 4.4 depicts the fluctuation of velocity with Peclet number (Pe). Velocity is observed to decrease as Pe increases. The Peclet number indicates the ratio of convective heat transfer to thermal diffusion. Higher Pe values diminish thermal diffusion effects, weakening the buoyancy-driven flow component and decreasing velocity.

Figure 4.5 shows how the Peclet number affects temperature distribution. The findings show that temperature rises with increasing Pe. Higher Peclet numbers correspond to lower thermal diffusion, which causes heat to accumulate within the fluid. As a result, the temperature boundary layer thickens and the overall temperature within the annulus rises.

## 5.0 Conclusion

This study provides an analytical evaluation of transient mixed convection flow in a vertical concentric annulus filled with porous material in the presence of a magnetic field. The governing momentum and energy equations were effectively converted into dimensionless form and solved analytically via perturbation and eigenfunction expansion techniques.

The findings show that critical dimensionless parameters have a significant impact on the flow and thermal behaviour within the annulus. The magnetic parameter (M) was discovered to drastically reduce fluid velocity due to the resistive Lorentz force, emphasizing its importance in flow control applications. In contrast, increases in the Grashof number (Gr) and Reynolds number (Re) increase fluid velocity, showing that buoyancy and inertial forces drive the flow.

The Peclet number (Pe) was shown to have conflicting impacts on velocity and temperature. While raising Pe lowers fluid velocity due to reduced thermal diffusion, it also enhances temperature distribution within the annulus. This highlights the close relationship between heat transfer and fluid mobility in porous structures.

In conclusion, the analytical model developed in this study provides understanding of the behaviour of mixed convection flow in porous annular systems. The findings are applicable to technical applications such as heat exchangers, geothermal systems, and cooling technologies, all of which require precise control over heat and flow. Furthermore, the study stresses the need of addressing coupled physical impacts when designing and optimizing porous media systems.

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