
MATHEMATICAL MODELLING OF COUPLED HEAT AND MASS TRANSFER IN CONVECTIVE–DIFFUSIVE TEXTILE DRYING

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Abstract

This paper describes a mathematical model for coupled heat and mass transfer in a convective-diffusion textile drying process. The model was developed to examine the simultaneous transport of heat and moisture through textile materials during industrial drying procedures. The governing partial differential equations for gas phase temperature, solid phase temperature, air moisture, and tissue moisture were developed using convection, diffusion, and evaporation concepts. Appropriate beginning and boundary conditions were applied, and the equations were translated into dimensionless form with appropriate scaling variables. Analytical solutions for the resulting system were obtained using the eigenfunction expansion technique and computed with MAPLE 17 software. The graphical results showed that moisture concentration decreases with drying time and distance, while temperature distributions vary significantly due to coupled heat and mass transfer effects. The study demonstrates that convective airflow enhances drying efficiency and provides useful information for improving industrial textile drying processes and energy management.

Keywords: Heat and Mass transfer, Textile Drying Process, Textile industries, Eigenfunction Expansion Technique, Textile Fibers

1. INTRODUCTION

Drying is one of the most significant procedures in textile manufacturing since Fibers, yarns, and fabrics often retain a large amount of moisture after the dyeing, washing, and finishing processes. The elimination of moisture is required to improve product quality, dimensional stability, durability, and storage conditions. Convective hot-air drying is widely used in the textile industry due to its efficiency and cheap operating costs. However, high heating when drying can damage textile fibres, particularly hygroscopic fabrics like wool, lowering their economic value and mechanical strength (Beirut, 2010).

The drying process employs simultaneous heat and mass transfer mechanisms, with thermal energy supplied to the wet material and moisture migrating from the textile inside to the surrounding environment. Drying activities take a lot of energy because water evaporation requires a high latent heat of vaporization. As a result, drying processes are recognized as among the most energy-intensive operations in the textile industry (Akyol *et al.* 2012). Various drying techniques, including infrared drying, microwave drying, vacuum drying, and hot-air convective drying, have been developed to improve drying efficiency and reduce energy consumption (Sonawane and Keste, 2016). Heat transfer happens when two locations have different temperatures, causing thermal energy to travel from one to the other. Heat transfer in textile drying systems is mostly accomplished via convection, which happens when hot air travels over the textile surface. Conduction and radiation may also contribute, depending on the drying configuration used. Moisture transport happens simultaneously via diffusion and convection mechanisms within the textile structure, as well as between the textile surface and the surrounding air (Haghi, 2011).

Textile materials' drying behaviour is extremely complex because it involves linked transient heat and mass transfer mechanisms. Moisture migrates from the inside of the textile material to the surface due to concentration gradients, whereas heat penetrates the material due to temperature gradients. Moisture evaporation at the textile surface has a considerable influence on temperature distribution within the material because latent heat is continuously used during evaporation (Perussello *et al.*, 2014). As a result, precise prediction of textile drying behaviour necessitates a thorough understanding of linked heat and mass transport phenomena.

Several academics have presented mathematical models for investigating textile drying systems. Lee *et al.* (2011) studied through-air drying processes for textile materials using coupled heat and mass transport equations. Sousa *et al.* (2012) investigated the drying properties of cotton fabrics in hot-air circumstances and discovered that drying rates are substantially impacted by air temperature and velocity. Similarly, Johann *et al.* (2014) constructed mathematical models for convective drying of textile materials using energy and mass balance equations, but they ignored diffusion effects in their formulation.

Many theoretical drying models in the literature rely on simplified assumptions, which may diminish prediction accuracy. According to Oumarou *et al.* (2015), several drying models assume a constant material temperature throughout the drying process, resulting in overestimation of drying rates. Other research looked primarily at diffusion mechanisms, ignoring evaporation and convection effects in the gas phase. However, evaporation is important during textile drying because it influences both moisture removal and temperature distribution inside the textile material (Perussello *et al.*, 2014). Due of rising operational expenses and global energy consumption, modern textile industries have prioritized energy efficiency. Sonawane and Keste (2016) underlined that increasing drying efficiency can

greatly reduce industrial energy consumption and environmental effect. Furthermore, Giovanni et al. (2014) evaluated industrial cotton yarn drying systems, emphasizing the need of optimizing drying conditions to reduce energy consumption while maintaining product quality.

Despite extensive investigations on textile drying processes, many existing models continue to ignore essential factors such as convection in the gas phase, diffusion inside textile materials, and evaporation during drying. These simplifications impair the accuracy of drying estimates under real-world industrial circumstances. As a result, more extensive mathematical models are required to fully describe coupled heat and mass movement in convective-diffusion textile drying processes (Johann *et al.*, 2014).

This research focuses on the analytical modelling of heat and mass transfer in a convective-diffusion textile drying process. The study's goal is to develop analytical solutions capable of forecasting moisture and temperature distributions within textile materials during drying using eigenfunction expansion approaches (Oumarou *et al.*, 2015).

2. Mathematical Formulation

A coupled heat and mass transfer model for the convective-diffusion textile drying process in textile industries was created. The formulation extended Johann *et al.*'s (2014) model by taking into account finite textile thickness and integrating diffusion and evaporation effects. The models were built on various simplifying assumptions, such as one-dimensional airflow parallel to the textile surface, perfect gas behaviour of air, constant textile volume, and evaporation occurring solely at the textile surfaces. The figure below depicts heat and mass diffusion in both the air and the textile material.

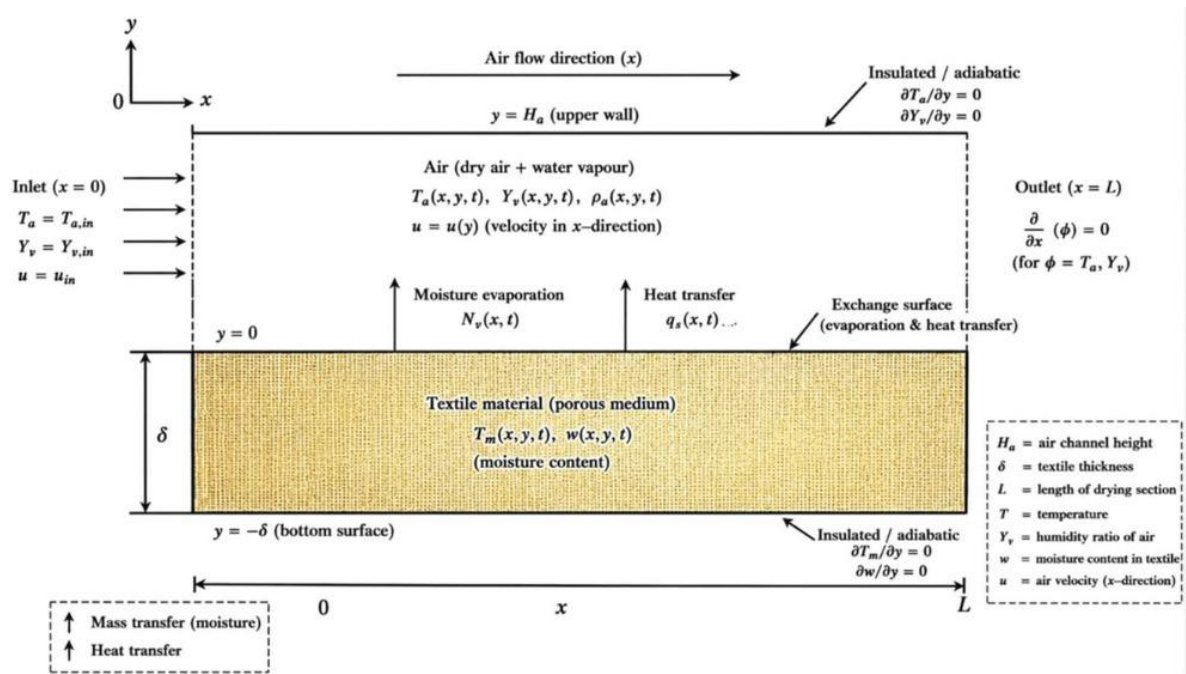


Figure 2.1 Flow Geometry and Physical model for convective–diffusive textile drying

Based on these assumptions, a system of coupled partial differential equations was formulated and presented as follows

diffusion textile drying process are;

$$\rho_g \left(\frac{\partial Y}{\partial t} + V \frac{\partial Y}{\partial z} \right) + Y \left(\frac{\partial \rho_g}{\partial t} + V \frac{\partial \rho_g}{\partial z} \right) = D_Y \frac{\partial^2 Y}{\partial z^2} + \frac{2L_s}{AT_g} K_f (Y_{eq} - Y) \quad (1)$$

$$\frac{\partial \rho_g}{\partial t} + V \frac{\partial \rho_g}{\partial z} = 0 \quad (2)$$

$$\rho_s \varepsilon_s \frac{\partial X}{\partial t} = D_X \frac{\partial^2 X}{\partial z^2} - 2K_f (Y_{eq} - Y) \quad (3)$$

$$\rho_g C_p \left(\frac{\partial T_g}{\partial t} + V \frac{\partial T_g}{\partial z} \right) = K_g \frac{\partial^2 T_g}{\partial z^2} - \frac{2h_c L_s}{AT_g} (T_g - T_s) + \frac{2L_s}{AT_g} \Delta H_s K_f (Y_{eq} - Y) \quad (4)$$

$$\rho_s \varepsilon_s C_p \frac{\partial T_s}{\partial t} = K_s \frac{\partial^2 T_s}{\partial z^2} + 2h_c (T_g - T_s) - 2\Delta H_s K_f (Y_{eq} - Y) \quad (5)$$

The initial and boundary conditions are formulated as:

$$\left. \begin{aligned} Y(z, 0) = Y_0, Y(0, t) = Y_{feed}, Y(L_s, t) = 0 \\ X(z, 0) = X_0, X(0, t) = 0, X(L_s, t) = 0 \\ T_g(z, 0) = T_{g0}, T_g(0, t) = T_{gfeed}, T_g(L_s, t) = 0 \\ T_s(z, 0) = T_{s0}, T_s(0, t) = T_{s0}, T_s(L_s, t) = T_{s0} \end{aligned} \right\} \quad (6)$$

Where;

z is the distance in space, t is the time, Y is the Absolute moisture of air, T_g is the Gas temperature (K), ρ_g is the Density of the dry gas, T_s is the Solid temperature, X is the Tissue moisture, V is the Velocity of the drying air, ρ_s is the Density of the dry solid, ε_s is the Textile thickness, K_f is the Mass transfer coefficient, Y_{eq} is the absolute equilibrium moisture of air, C_p is the heat capacity at constant pressure, K_g is the thermal conductivity of the textile material, K_s is the thermal conductivity of the solid material, h_c is the convective heat transfer coefficient $kJ/(m^2 km)$, L_s is the length of textile (m), AT_g is the transversal area of the air feed box, ΔH_s is the Desorption enthalpy (kJ/kg) water, C_{p_v} is the specific heat of a steam in the (kJ/kg) water solid at constant pressure, D_X is the diffusion coefficient tissue moisture, D_Y is the diffusion coefficient of absolute moisture of air, Δh_0 is the vaporization energy of water at $0^\circ C$, $C_{p_{ws}}$ is the specific heat of water in solid at constant pressure, u is the reference velocity, ρ_{g0} is the initial density of ambient air, Y_0 is the Humidity.

2.1 Non - dimensionalisation

To we have the model in a non- dimensionless form, the following dimensionless variables for space, time, temperature, condensed fluid concentration and oxygen concentration are introduced:

$$\left. \begin{aligned} z' = \frac{z}{L_s}, t' = \frac{ut}{L_s}, \theta = \frac{T_g - T_{g0}}{T_{gfeed} - T_{g0}}, \phi = \frac{T_s - T_{s0}}{T_{gfeed} - T_{s0}}, \\ Y' = \frac{Y}{Y_0}, X' = \frac{X}{X_0}, \rho = \frac{\rho_g}{\rho_{g0}}, V' = \frac{V}{u} \end{aligned} \right\} \quad (7)$$

Substituting equation (7) into equations (1) – (6) and processing it, after dropping the prime, we obtained:

$$\frac{\partial Y}{\partial t} + V \frac{\partial Y}{\partial z} + Y \left(\frac{\partial \rho}{\partial t} + V \frac{\partial \rho}{\partial z} \right) = \frac{1}{P_{em1}} \frac{\partial^2 Y}{\partial z^2} + \sigma(a - Y) \quad (8)$$

$$\frac{\partial X}{\partial t} = \frac{1}{P_{em2}} \frac{\partial^2 X}{\partial z^2} - \sigma_1(a - Y) \quad (9)$$

$$\rho \left(\frac{\partial \theta}{\partial t} + V \frac{\partial \theta}{\partial z} \right) = \frac{1}{P_{e1}} \frac{\partial^2 \theta}{\partial z^2} - \beta(\theta - b\phi + c) + \beta_1(a - Y) \quad (10)$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{P_{e2}} \frac{\partial^2 \phi}{\partial z^2} + \alpha(\theta - b\phi + c) - \alpha_1(a - Y) \quad (11)$$

$$\left. \begin{aligned} Y(z, 0) = 1, Y(0, t) = q, Y(1, t) = 0, \\ X(z, 0) = 1, X(0, t) = 0, X(1, t) = 0, \\ \theta(z, 0) = 0, \theta(0, t) = 1, \theta(1, t) = 0, \\ \phi(z, 0) = 0, \phi(0, t) = 0, \phi(1, t) = 0, \end{aligned} \right\} \quad (12)$$

Where;

$P_{em1} = \frac{L_s \rho_{g0} u}{D_Y}$ is the peclet mass number for moisture of air

$\sigma = \frac{2L_s^2 K_f}{AT_g \rho_{g0} u}$ is the mass transfer coefficient for gas,

$P_{em2} = \frac{L_s \rho_s \varepsilon_s u}{D_X}$ is the peclet mass number for tissue moisture,

$P_{e1} = \frac{\rho_{g0} C_p u L_s}{K_g}$ is the peclet energy number for textile material,

$\sigma_1 = \frac{2K_f L_s Y_0}{\rho_s \varepsilon_s u X_0}$ is the mass transfer coefficient for solid,

$\beta = \frac{2h_c L_s}{AT_g \rho_{g0} C_p}$ is the convective heat transfer of gas parameter

$c = \left(\frac{T_{g0} - T_{s0}}{T_{gfeed} - T_{g0}} \right)$, $\beta_1 = \frac{2L_s^2 \Delta H_s K_f Y_0}{\rho_{g0} C_p u (T_{gfeed} - T_{g0})}$ is the desorption enthalpy parameter for gas

$P_{e2} = \frac{\rho_s \varepsilon_s C_p u L_s}{K_s}$ is the peclet energy number,

$\alpha = \frac{2h_c}{\rho_s \varepsilon_s C_p u} \left(\frac{T_{gfeed} - T_{g0}}{T_{gfeed} - T_{s0}} \right)$ is the coefficient of heat transfer for solid

$\alpha_1 = \frac{2\Delta H_s K_f L_s Y_0}{\rho_s \varepsilon_s C_p u (T_{gfeed} - T_{s0})}$ is the desorption enthalpy parameter for solid

3.2 Analytical Solution

Let $0 < V \ll 1$ and $b = mV$ such that

$$\left. \begin{aligned} Y &= Y_0 + VY_1 \\ X &= X_0 + VX_1 \\ \theta &= \theta_0 + V\theta_1 \\ \phi &= \phi_0 + V\phi_1 \end{aligned} \right\} \quad (13)$$

Substitute equation (13) into equations (8) - (12) and processes, we obtained

$$\left. \begin{aligned} \frac{\partial Y_0}{\partial t} &= \frac{1}{P_{em1}} \frac{\partial^2 Y_0}{\partial z^2} + \sigma(a - Y_0) \\ Y_0(z, 0) &= 1, Y_0(0, t) = q, Y_0(1, t) = 0 \end{aligned} \right\} \quad (14)$$

$$\left. \begin{aligned} \frac{\partial X_0}{\partial t} &= \frac{1}{P_{em2}} \frac{\partial^2 X_0}{\partial z^2} - \sigma_1(a - Y_0) \\ X_0(z, 0) &= 1, X_0(0, t) = 0, X_0(1, t) = 0 \end{aligned} \right\} \quad (15)$$

$$\left. \begin{aligned} \rho \frac{\partial \theta_0}{\partial t} &= \frac{1}{P_{e1}} \frac{\partial^2 \theta_0}{\partial z^2} - \beta(\theta_0 + c) + \beta_1(a - Y_0) \\ \theta_0(z, 0) &= 0, \theta_0(0, t) = 1, \theta_0(1, t) = 0 \end{aligned} \right\} \quad (16)$$

$$\left. \begin{aligned} \frac{\partial \phi_0}{\partial t} &= \frac{1}{P_{e2}} \frac{\partial^2 \phi_0}{\partial z^2} + \alpha(\theta_0 + c) - \alpha_1(a - Y_0) \\ \phi_0(z, 0) &= 0, \phi_0(0, t) = 0, \phi_0(1, t) = 0 \end{aligned} \right\} \quad (17)$$

$$\left. \begin{aligned} \frac{\partial Y_1}{\partial t} &= \frac{1}{P_{em1}} \frac{\partial^2 Y_1}{\partial z^2} - \frac{\partial Y_0}{\partial z} - \sigma Y_1 \\ Y_1(z, 0) &= 0, Y_1(0, t) = 0, Y_1(1, t) = 0 \end{aligned} \right\} \quad (18)$$

$$\left. \begin{aligned} \frac{\partial X_1}{\partial t} &= \frac{1}{P_{em2}} \frac{\partial^2 X_1}{\partial z^2} + \sigma_1 Y_1 \\ X_1(z, 0) &= 0, X_1(0, t) = 0, X_1(1, t) = 0 \end{aligned} \right\} \quad (19)$$

$$\rho \frac{\partial \theta_1}{\partial t} = \frac{1}{P_{e1}} \frac{\partial^2 \theta_1}{\partial z^2} - \rho \frac{\partial \theta_0}{\partial t} - \beta(\theta_1 - m\varphi_0) - \beta_1 Y_1, \quad (20)$$

$$\left. \begin{aligned} \theta_1(z, 0) &= 0, \theta_1(0, t) = 0, \theta_1(1, t) = 0 \\ \frac{\partial \varphi_1}{\partial t} &= \frac{1}{P_{e2}} \frac{\partial^2 \varphi_1}{\partial z^2} + \alpha(\theta_1 - m\varphi_0) + \alpha_1 Y_1 \\ \varphi_1(z, 0) &= 0, \varphi_1(0, t) = 0, \varphi_1(1, t) = 0 \end{aligned} \right\} \quad (21)$$

Using eigenfunction expansion technique, we obtain solution for equations (14) – (21) as follows:

$$Y_0(z, t) = (q - qz) + \sum_{n=1}^{\infty} ((F_1 + F_2)e^{-c_1 t}) \sin(n\pi z) \quad (22)$$

$$X_0(z, t) = \sum_{n=1}^{\infty} (P_7(1 - e^{-c_2 t}) + \sigma_1 \sum_{n=1}^{\infty} (P_8(e^{-c_1 t} - e^{-c_2 t}) + P_9(1 - e^{-c_2 t}))) \sin(n\pi z) \quad (23)$$

$$\begin{aligned} \theta_0(z, t) &= (1 - z) + \sum_{n=1}^{\infty} \left(P_{13} \sum_{n=1}^{\infty} \left(\frac{F_2}{c_3 + c_1} e^{-c_1 t} + \frac{F_1}{c_3} \right) - \frac{P_{14}}{c_3} (1 - e^{c_3 t}) \right) \sin(n\pi z) \\ \phi_0(z, t) &= \sum_{n=1}^{\infty} \left(P_{19}(e^{-c_4 t} - 1) + \right. \\ &\quad \left. \sum_{n=1}^{\infty} P_{17} \frac{1 - e^{-c_4 t}}{c_4} - \frac{1 - e^{-c_4 t}}{c_4} + \frac{c_3 e^{-c_4 t} + c_3 t - 1}{c_3} + \frac{P_{13} \sum_{n=1}^{\infty} P_{24} e^{-c_4 t} - 1}{c_3} + \frac{P_{22} e^{-c_4 t} - c_1 t - 1}{c_3} + \frac{P_{23} e^{-c_4 t} - 1}{c_3} \right) \sin(n\pi z) \end{aligned} \quad (24)$$

$$Y_1(z, t) = \sum_{n=1}^{\infty} P_{26} (1 - e^{-c_1 t}) \sin(n\pi z) \quad (25)$$

$$X_1(z, t) = \sum_{n=1}^{\infty} \sigma_1 \left(\sum_{n=1}^{\infty} P_{29} e^{-c_2 t} - P_{28} e^{-c_1 t} + P_{27} \right) \sin(n\pi z) \quad (26)$$

$$\begin{aligned} \theta_1(z, t) &= \sum_{n=1}^{\infty} \left(P_{11} m \sum_{n=1}^{\infty} P_{29} (e^{-c_4 t} - e^{-c_3 t}) - P_{30} (1 - e^{-c_3 t}) \right. \\ &\quad \left. + \sum_{n=1}^{\infty} \left(P_{31} (e^{c_4 t} - e^{-c_3 t}) - P_{32} (1 - e^{-c_3 t}) - P_{33} (e^{(c_4 + c_3)t} - e^{-c_3 t}) - \right) + P_{13} \right) \sin(n\pi z) \end{aligned}$$

$$\phi_1(z, t) = \alpha \sum_{n=1}^{\infty} \left(\sum_{m=1}^{\infty} \left(\begin{aligned} &P_{29} \left(t e^{-c_4 t} - \frac{e^{(-c_3)t}}{(c_4 - c_3)} + \frac{e^{-c_4 t}}{(c_4 - c_3)} \right) \\ &- P_{30} \left(\frac{1}{c_4} - \frac{e^{-c_4 t}}{c_4} - \frac{e^{(-c_3)t}}{(c_4 - c_3)} + \frac{e^{-c_4 t}}{(c_4 - c_3)} \right) \\ &\left(\begin{aligned} &P_{31} \left(\frac{e^{c_4 t}}{2c_4} - \frac{e^{-c_4 t}}{2c_4} - \frac{e^{(-c_3)t}}{(c_4 - c_3)} + \frac{e^{-c_4 t}}{(c_4 - c_3)} \right) \\ &- P_{32} \left(\frac{1}{c_4} - \frac{e^{-c_4 t}}{c_4} - \frac{e^{(-c_3)t}}{(c_4 - c_3)} + \frac{e^{-c_4 t}}{(c_4 - c_3)} \right) \\ &- P_{33} \left(\frac{e^{(c_4+c_3)t}}{(2c_4+c_3)} - \frac{e^{-c_4 t}}{(2c_4+c_3)} - \frac{e^{(-c_3)t}}{(c_4-c_3)} + \frac{e^{-c_4 t}}{(c_4-c_3)} \right) \\ &- P_{34} \left(\frac{1}{c_4} - \frac{e^{-c_4 t}}{c_4} - \frac{e^{(-c_3)t}}{(c_4 - c_3)} + \frac{e^{-c_4 t}}{(c_4 - c_3)} \right) \end{aligned} \right) \\ &+ P_{13} \sum_{n=1}^{\infty} \left(\begin{aligned} &P_{35} \left(t - \frac{e^{(-c_3)t}}{(c_4 - c_3)} + \frac{e^{-c_4 t}}{(c_4 - c_3)} \right) - P_{36} \left(\frac{1}{c_4} - \frac{e^{-c_4 t}}{c_4} - \frac{e^{(-c_3)t}}{(c_4 - c_3)} + \frac{e^{-c_4 t}}{(c_4 - c_3)} \right) \\ &+ P_{37} \left(\frac{e^{(c_4-c_1)t}}{(2c_4-c_1)} - \frac{e^{-c_4 t}}{(2c_4-c_1)} - \frac{e^{(-c_3)t}}{(c_4-c_3)} + \frac{e^{-c_4 t}}{(c_4-c_3)} \right) \\ &- P_{38} \left(\frac{1}{c_4} - \frac{e^{-c_4 t}}{c_4} - \frac{e^{(-c_3)t}}{(c_4 - c_3)} + \frac{e^{-c_4 t}}{(c_4 - c_3)} \right) \\ &+ P_{39} \left(\frac{e^{c_4 t}}{2c_4} - \frac{e^{-c_4 t}}{2c_4} - \frac{e^{(-c_3)t}}{(c_4 - c_3)} + \frac{e^{-c_4 t}}{(c_4 - c_3)} \right) - P_{40} \left(\frac{1}{c_4} - \frac{e^{-c_4 t}}{c_4} - \frac{e^{(-c_3)t}}{(c_4 - c_3)} + \frac{e^{-c_4 t}}{(c_4 - c_3)} \right) \\ &- P_{13} \sum_{n=1}^{\infty} \left(\begin{aligned} &P_{41} \left(\frac{1}{c_4} - \frac{e^{-c_4 t}}{c_4} - \frac{e^{(-c_3)t}}{(c_4 - c_3)} + \frac{e^{-c_4 t}}{(c_4 - c_3)} \right) \\ &- P_{42} \left(\frac{e^{(-c_1)t}}{(c_4 - c_1)} - \frac{e^{-c_4 t}}{(c_4 - c_1)} - \frac{e^{(-c_3)t}}{(c_4 - c_3)} + \frac{e^{-c_4 t}}{(c_4 - c_3)} \right) \end{aligned} \right) \end{aligned} \right) \sin(n\pi z) \end{aligned} \right) \\ - \alpha m \left(\sum_{n=1}^{\infty} P_{19} \left(t - \frac{1}{c_4} + \frac{e^{-c_4 t}}{c_4} \right) \right. \\ \left. + \sum_{n=1}^{\infty} P_{17} \left(\begin{aligned} &\left(\frac{1}{c_4} \left(\frac{e^{c_4 t}}{2c_4} - \frac{e^{-c_4 t}}{2c_4} - \frac{1}{c_4} + \frac{e^{-c_4 t}}{c_4} \right) - \frac{1}{(c_4+c_3)} \left(\frac{e^{(c_4+c_3)t}}{(2c_4+c_3)} - \frac{e^{-c_4 t}}{(2c_4+c_3)} - \frac{1}{c_4} + \frac{e^{-c_4 t}}{c_4} \right) \right) \right. \\ &+ P_{13} \sum_{n=1}^{\infty} P_{24} \left(\frac{e^{c_4 t}}{2c_4} - \frac{e^{-c_4 t}}{2c_4} - \frac{1}{c_4} + \frac{e^{-c_4 t}}{c_4} \right) \\ &+ P_{22} \left(\frac{e^{(c_4-c_1)t}}{(2c_4-c_1)} - \frac{e^{-c_4 t}}{(2c_4-c_1)} - \frac{1}{c_4} + \frac{e^{-c_4 t}}{c_4} \right) + P_{23} \left(\frac{e^{c_4 t}}{2c_4} - \frac{e^{-c_4 t}}{2c_4} - \frac{1}{c_4} + \frac{e^{-c_4 t}}{c_4} \right) \end{aligned} \right) \right) \\ \left. + \alpha_1 \left(\sum_{n=1}^{\infty} P_{26} \left(\frac{1}{c_4} - \frac{e^{-c_4 t}}{c_4} - \frac{e^{(-c_3)t}}{(c_4 - c_3)} + \frac{e^{-c_4 t}}{(c_4 - c_3)} \right) \right) \right) \end{aligned} \right) \quad (27)$$

Where;

$$\begin{aligned} c_1 &= \alpha - k \left(\frac{n\pi}{L} \right)^2, c_2 = \frac{(n\pi)^2}{P_{em2}}, c_3 = P_{11} - \frac{(n\pi)^2}{\rho P_{e1}}, c_4 = \frac{(n\pi)^2}{P_{e2}}, F_1 = \frac{2\sigma((a-q)-a(-1)^n)}{n\pi c_1}, \\ F_2 &= P_1 - F_1, P_1 = \frac{2(1-(-1)^n)}{n\pi}, P_4 = \sigma_1 a - \sigma_1 q, P_5 = \sigma_1 q, P_6 = -\frac{2P_4(1-(-1)^n)}{n\pi} - \frac{2P_5(-1)^n}{n\pi}, \\ P_7 &= \frac{P_6}{c_2}, P_8 = \frac{F_2}{c_2 - c_1}, P_9 = \frac{F_1}{c_2}, P_{10} = \frac{\beta_1 a - \beta c}{\rho}, P_{11} = \frac{\beta}{\rho}, P_{12} = \frac{\beta + \beta_1 q}{\rho}, P_{13} = \frac{\beta_1}{\rho}, \\ P_{14} &= 2(P_{10} - P_{12}) \left(\frac{1-(-1)^n}{n\pi} \right) - \frac{2P_{12}(-1)^n}{n\pi} \\ P_{15} &= \frac{F_2}{c_3 + c_1}, P_{16} = \frac{F_1}{c_3}, P_{17} = \frac{P_{14}}{c_3}, P_{18} = \frac{2(\alpha c - \alpha_1 a + \alpha - \alpha_1 q)}{n\pi} - \frac{2(\alpha - \alpha_1 q)}{n\pi}, P_{19} = \frac{P_{18}}{c_4}, P_{20} = \frac{P_{15}}{c_4 - c}, P_{21} = \\ \frac{P_{16}}{c_4}, P_{22} &= \frac{F_2}{c_4 - c_1}, P_{23} = \frac{F_1}{c_4}, P_{24} = P_{20} + P_{21}, P_{25} = -\frac{2q(1-(-1)^n)}{(n\pi)}, P_{26} = \frac{P_{25}}{c_1}, P_{27} = \frac{P_{26}}{c_2}, P_{28} = \\ \frac{P_{26}}{c_2 - c_1}, P_{29} &= P_{28} - P_{27}, P_{30} = \frac{P_{19}}{(c_3 - c_4)}, P_{31} = \frac{P_{19}}{c_3}, P_{32} = \frac{P_{17}}{c_4(c_3 + c_4)}, P_{33} = \frac{P_{17}}{c_4 c_3}, P_{34} = \frac{1}{(c_4 + c_3)(c_4 + 2c_3)} \end{aligned}$$

The computation of equations (22) – (27) were done using computer symbolic algebraic

package (MAPLE 17).

4.0 Result and Discussion

Heat and mass transfer in a connective-diffusion textile drying were simulated analytically using eigenfunction expansion technique. Analytical solution given by equations (22) – (27) are computed using the computer symbolic algebraic package MAPLE 17. The numerical results are discussed from physical point of view from the graphical illustrations shown in figure 4.1 to 4.16 below:

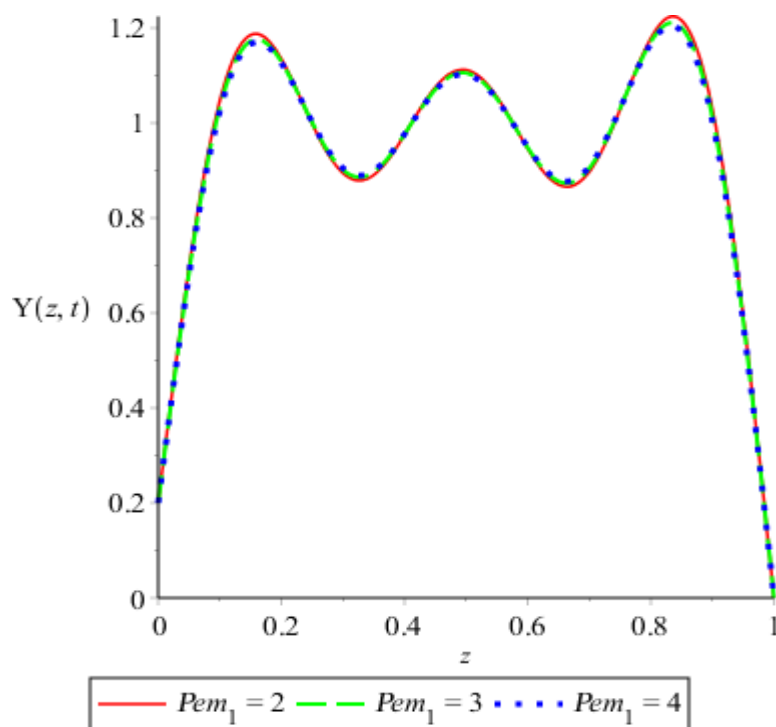


Figure 4.1: Graph of absolute moisture of air $Y(z, t)$ against distance z .

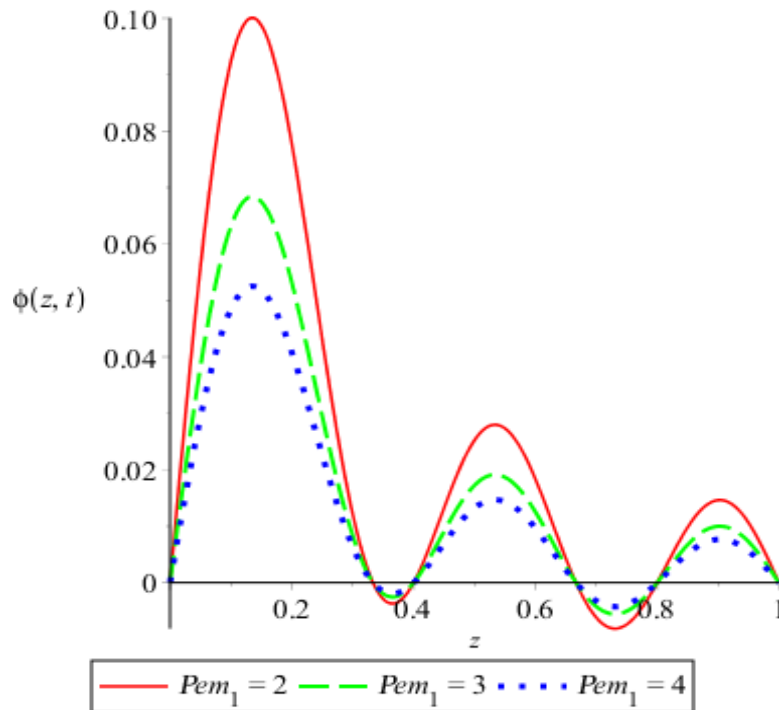


Figure 4.2: Graph of solid phase temperature $\phi(z,t)$ against distance z for different value of Peclet mass number for moisture of air.

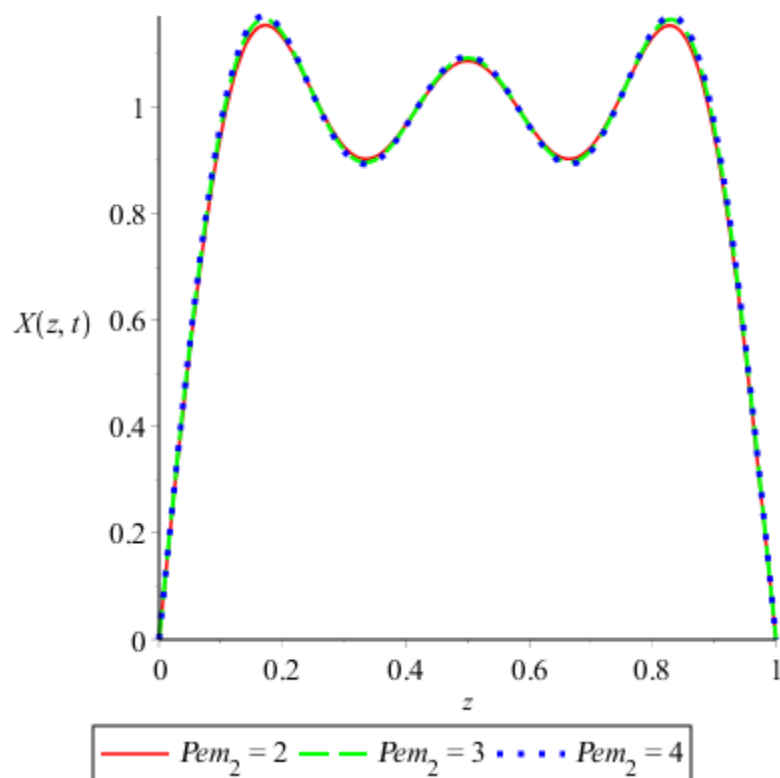


Figure 4.3: Graph of tissue moisture $X(z,t)$ against distance z .

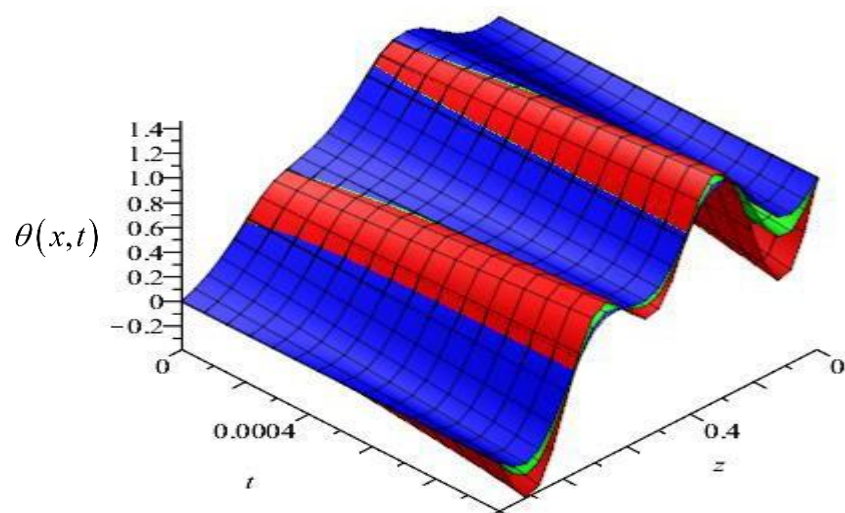


Figure 4.4: Graph of gas phase temperature against distance z and time t .

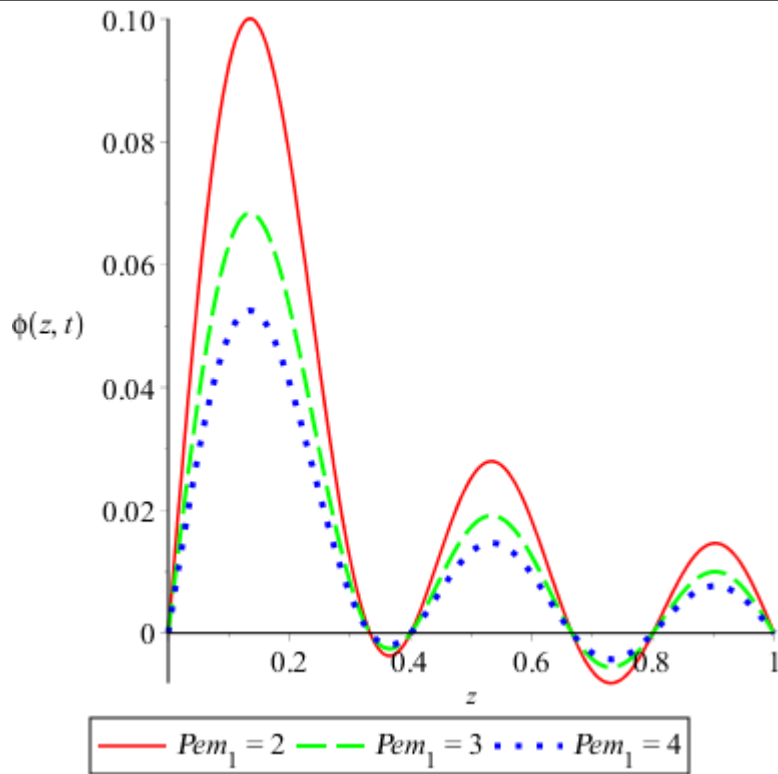


Figure 4.5: Graph of solid phase temperature $\phi(z,t)$ against distance z .

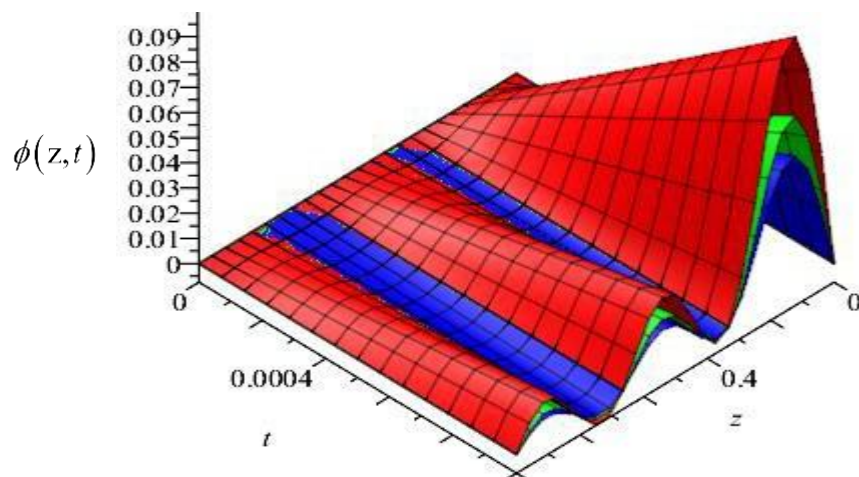


Figure 4.6: Graph of solid phase temperature $\phi(z,t)$ against distance z and time t .

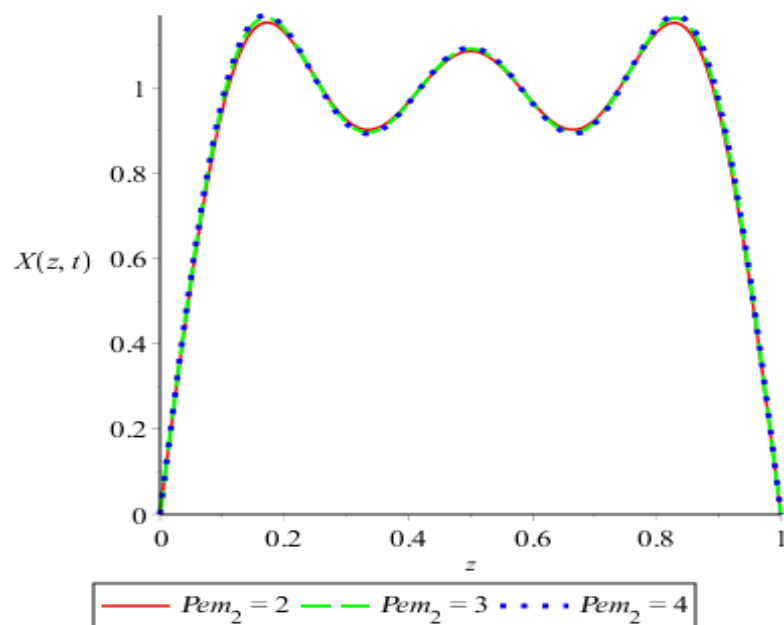


Figure 4.7: Graph of tissue moisture $X(z,t)$ against distance z .

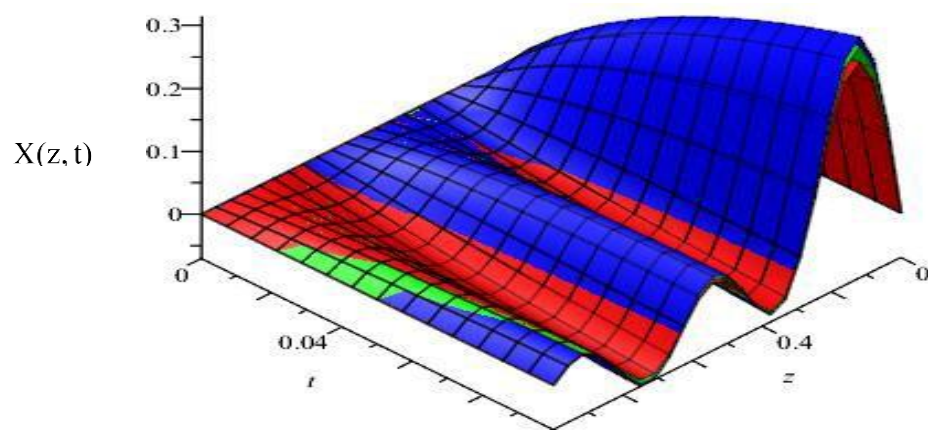


Figure 4.8: Graph of tissue moisture $X(z,t)$ against distance z and time t .

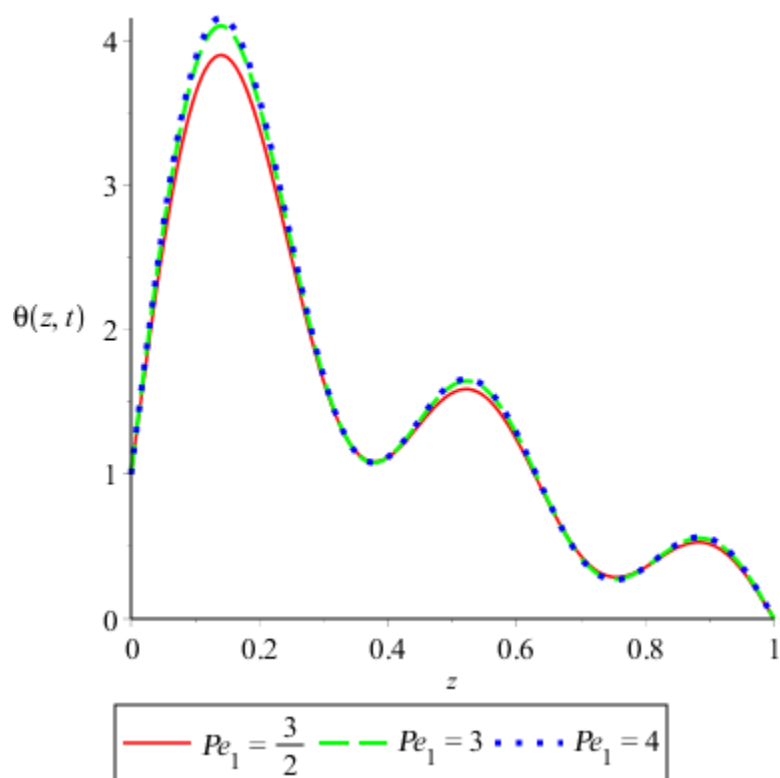


Figure 4.9: Graph of gas phase temperature $\theta(z, t)$ against distance z

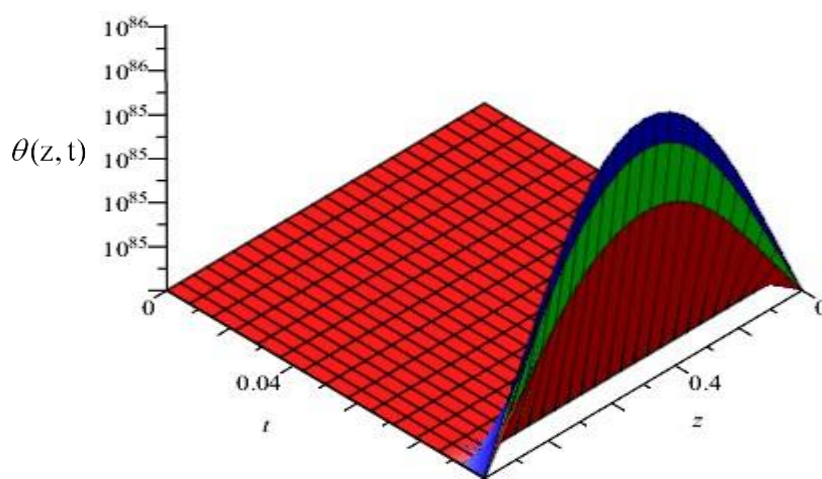


Figure 4.10: Graph of gas phase temperature $\theta(z, t)$ against distance z and time t .

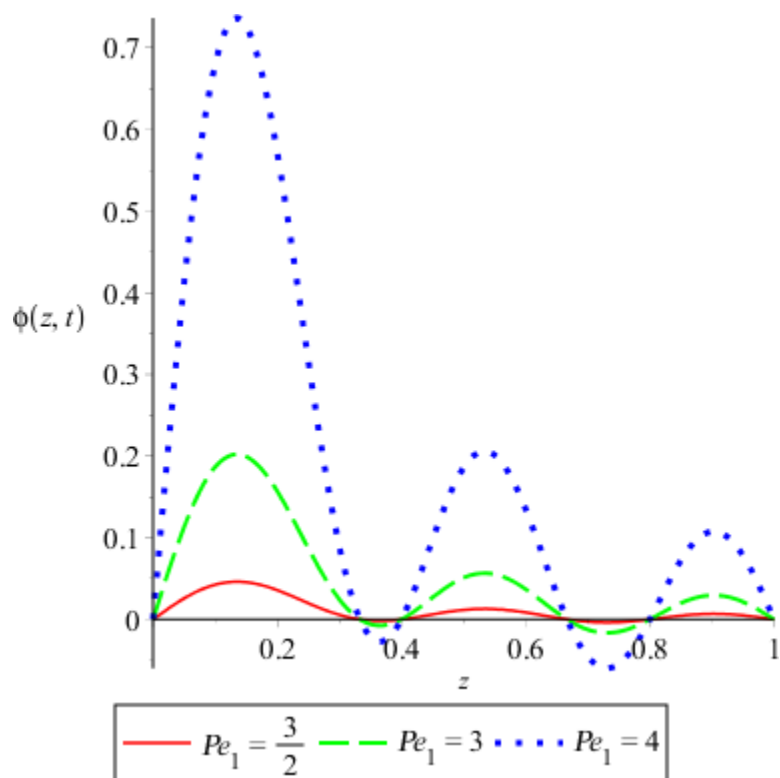


Figure 4.11: Graph of solid phase temperature $\phi(z, t)$ against distance z .

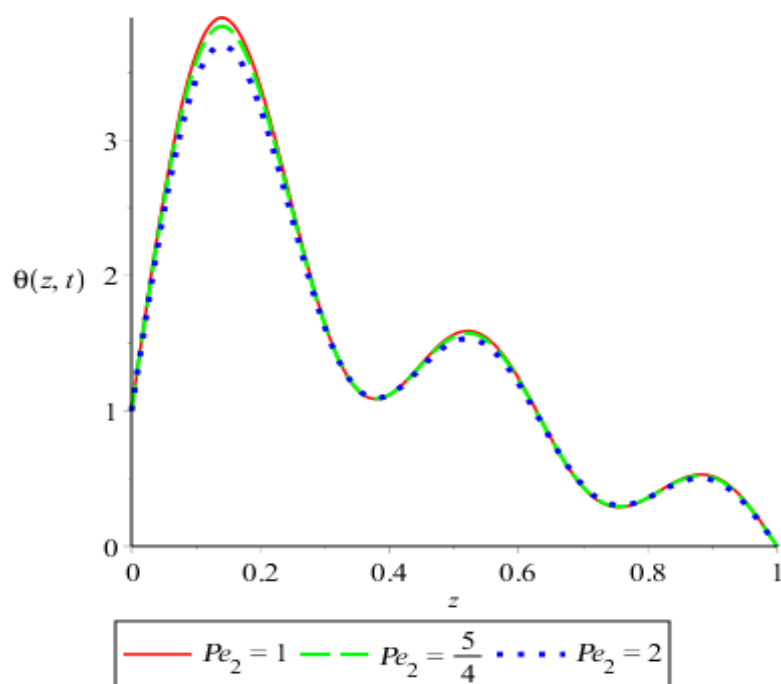


Figure 4.12: Graph of gas phase temperature $\theta(z, t)$ against distance z .

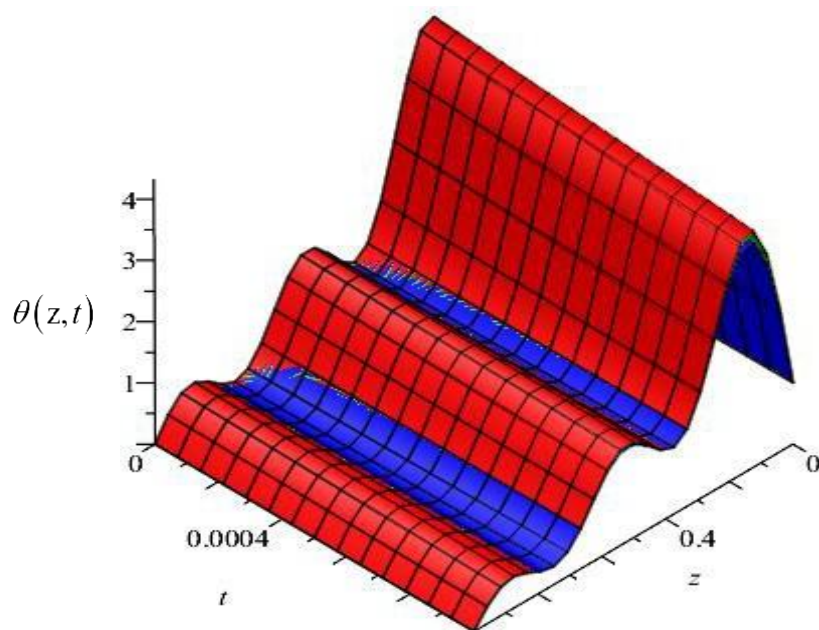


Figure 4.13: Graph of gas phase temperature $\theta(z, t)$ against distance z and time t .

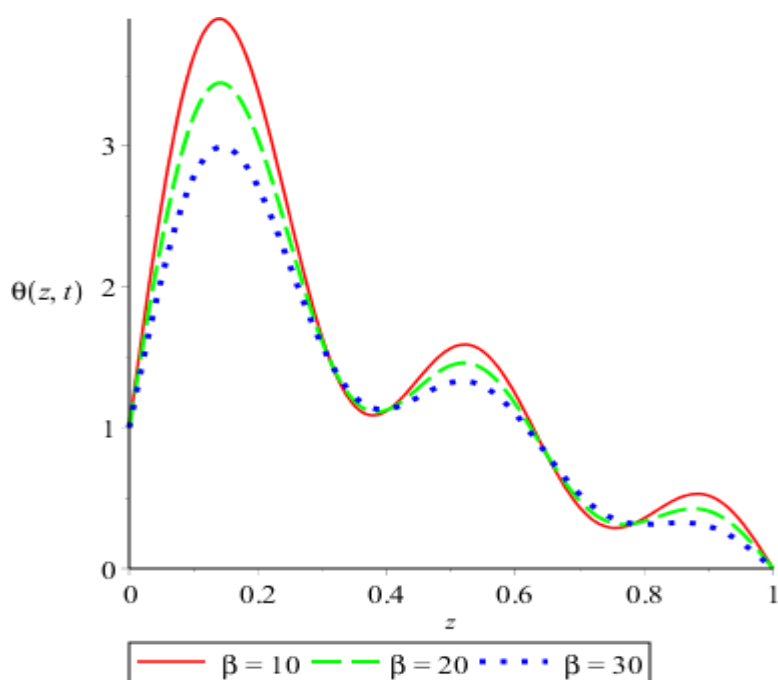


Figure 4.14: Graph of gas phase temperature $\theta(z, t)$ against distance z .

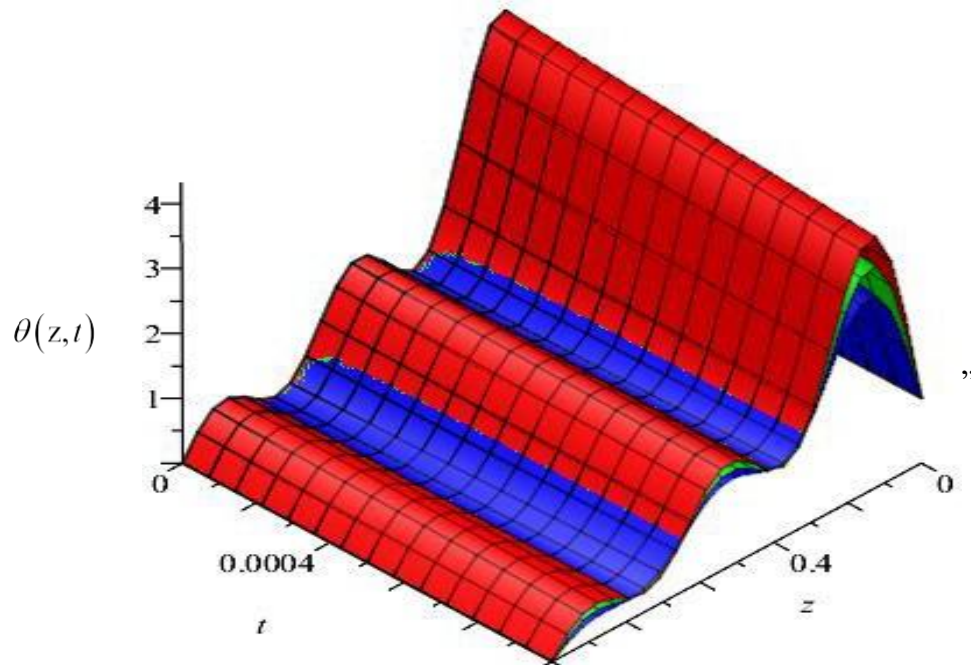


Figure 4.15: Graph of gas phase temperature $\theta(z, t)$ against distance z and time t

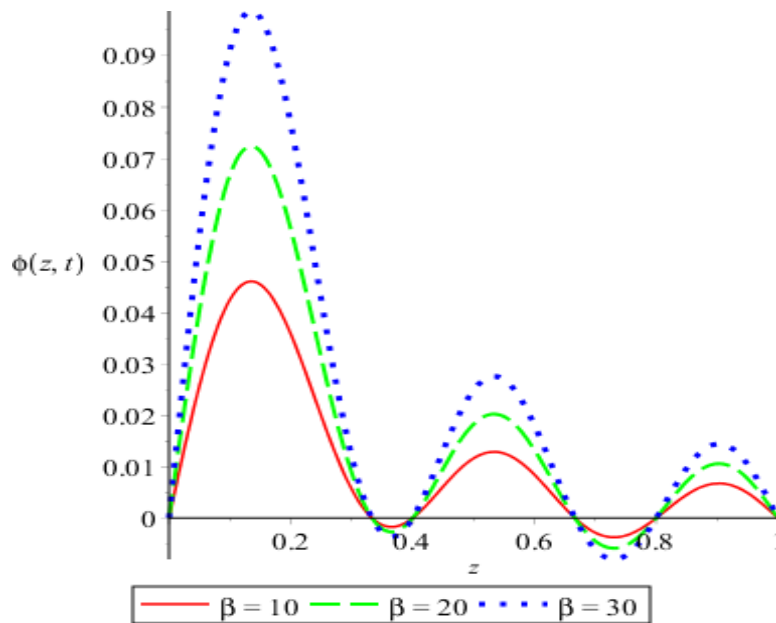


Figure 4.16: Graph of solid phase temperature $\phi(z,t)$ against distance z .

The graphical results presented in this study illustrate the behaviour of heat and mass transfer during the convective-diffusion drying of textile materials. The profiles explain the variation of moisture concentration and temperature distributions in both the gas and solid phases as drying progresses.

Figure 4.1: Absolute Moisture of Air $Y(z,t)$ Against Distance

The graph depicts how the absolute moisture of air steadily decreases along the drying chamber distance. This phenomenon arises as a result of moisture being continually transported from the textile material into the flowing hot air stream. At the entrance, the air has a lower moisture content, but as it runs over the wet cloth, evaporation increases the moisture carried by the air. The profile depicts how convective transport removes moisture from the textile surface.

Figure 4.2: Solid Phase Temperature for Different Peclet Mass Numbers

This graph depicts the effect of the Peclet mass number on the solid phase temperature distribution. Higher Peclet numbers indicate stronger convective transport compared to diffusion. The graph shows that raising the Peclet mass number improves heat penetration into the textile material, which leads to greater solid temperatures. This means that convective airflow has a major impact on the heating rate during drying.

Figures 4.3 and 4.7: Tissue Moisture $X(z,t)$ Against Distance

This behavior confirms that moisture migrates from the interior of the textile to the surface and is removed by evaporation. Initially, the textile contains high moisture concentration, but continuous heat supply accelerates evaporation and diffusion processes, causing the moisture profile to steadily decline. The result agrees with practical textile drying behavior, where moisture content reduces progressively until equilibrium conditions are reached.

Figures 4.4, 4.9, 4.12 and 4.14: Gas Phase Temperature $\theta(z, t)$

The gas phase temperature profiles vary with both distance and time. The temperature normally drops along the airflow direction because moisture evaporation consumes some of the thermal energy supplied by the hot air. The fall in gas temperature indicates the close relationship between heat and mass transfer mechanisms. As the gas dries, it loses thermal energy to the textile and latent heat of vaporization.

Figures 4.5, 4.6, 4.11 and 4.16: Solid Phase Temperature $\phi(z, t)$

The solid phase temperature steadily rises over time due to heat absorption from the hot air stream. Initially, the textile temperature is modest since most of the given energy is used for evaporation. As moisture content drops, less latent heat is required, allowing the textile temperature to increase even further. The figures also show that temperature distribution becomes more uniform with longer drying times, implying enhanced thermal equilibrium within the textile material.

Figures 4.8, 4.13 and 4.15: Combined Effects of Distance and Time

These three-dimensional profiles show how moisture and temperature affect both spatial distance and drying time. The data indicate that: During drying, moisture concentration reduces over time, gas and solid temperatures fluctuate, and heat transfer and moisture diffusion are closely related. The surfaces exhibit transient drying behavior, indicating that the process is time-dependent and governed by both convection and diffusion mechanisms. In summary, the graphical results support the validity of the proposed convective diffusion drying method. The model predicts that:

Moisture evaporation improves with airflow and drying time. Heat transfer from the gas phase leads to evaporation. Temperature gradients significantly impact moisture dispersion. Convective effects dominate the drying process. Furthermore, the findings show that enhancing convective movement improves drying efficiency, but evaporation has a substantial impact on temperature distribution in both the air and textile materials. These findings are critical for optimizing industrial textile drying systems to save energy while preserving product quality.

Significance of the Study

The findings of this work are significant because they improve our understanding of the linked heat and mass transfer processes involved in convective-diffusion textile drying. The created mathematical model aids in predicting temperature and moisture distributions inside textile materials during drying, hence enhancing drying efficiency, lowering energy consumption, and limiting fabric damage caused by overheating. The analytical solutions also help to optimize industrial drying parameters including airflow rate, drying temperature, and drying time.

The practical applications of this work include the design and improvement of textile drying equipment, energy management in textile industries, and enhancement of product quality during fabric processing. The model can also assist engineers in developing automated drying control systems for industrial applications.

The beneficiaries of this research include textile manufacturing industries, process engineers, researchers in heat and mass transfer, energy management experts, equipment designers, and academic institutions involved in industrial and applied mathematics research.

Conclusion.

This study developed and investigated a mathematical model of coupled heat and mass transfer in a convective-diffusion textile drying process. The governing equations for moisture diffusion, heat transfer, evaporation, and convection within the textile material and surrounding air were developed and converted into a dimensionless format. Analytical solutions were obtained using the eigenfunction expansion technique, and the results were graphically presented. The graphical results showed that moisture content gradually declines with distance and drying time, but gas and solid phase temperatures vary dramatically due to evaporation and heat transfer effects. The study also demonstrated that convective airflow and Peclet numbers have a significant impact on drying efficiency and temperature dispersion throughout the textile material. The created model provides a valid framework for predicting textile drying behaviour, which may be used to optimize industrial drying systems in order to reduce energy consumption, increase drying performance, and maintain textile product quality.

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