Calculation of Momentum Distributions of ^{18}C Core Fragment From ^{19}C + ^{9}Be Reaction Using Glauber Theory

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ABSTRACT

The longitudinal momentum distributions of ^{18}**C** *core fragment from the reaction* ^{19}**C+**^{9}**Be,** *was computed in the framework of Glauber theory using the CSC_GM program code which provides in the first order the internal momentum distribution of removed neutron. The CSC_GM code is a Fortran 90 program which was modified and run on Linux operating system. The projectile nucleus is assumed to have the structure of a core plus a valence neutron. The result, found to agree with the experimental data, show that* ^{18}**C,** *exhibits halo structure.*

Keywords: *Glauber model, One-nucleon halo, Momentum distribution.*

1.0 Introduction

The early interest in momentum distributions came from the studies of nuclear halo states, for which the narrow momentum distributions in a qualitative way revealed the large spatial extension of the halo wave function. It was shown by *Bertulani and Mc Voy (1992)* that the longitudinal component of the momentum gave the most appropriate information on the intrinsic properties of the halo nuclei and that it was insensitive to details of the collision and the size of the target. Different sophisticated models had been developed to describe the properties of halo nuclei by calculating their reaction observables (such as momentum distributions). These models include the Glauber model, which has been applied by many authors to determine these observables over a wide range of energies (Cortina-Gil et al., 2001). In this paper momentum distribution of ^{18}C core fragment from the ^{19}C + ^{9}Be reaction system is calculated in the framework of the Glauber Theory using the CSC_GM code because it can account for a significant part of breakup effects which play an important role in the reaction of a weakly bound nucleus and revealing a nuclear structure of a halo nucleus (Glauber et al., 1959). The reaction systems are described as $^{19}C + {}^{9}Be \rightarrow {}^{18}C + {}^{9}Be + n$. The projectile nucleus (^{19}C) is assumed to have structure of a core nucleus (^{18}C) plus a valence neutron. The model gives the nucleus–nucleus interaction in terms of interaction between the constituent nucleons with a given density distribution. The model is a semi-classical one picturing the nuclear collision in the impact parameter representation and thus works well at high energy (Ozawa et al., 2001).

2.0 Theoretical Background

Let's consider the reaction between a projectile nucleus P with a target nucleus T, at the first stage of the reaction, the projectile nucleus in the ground state is described with an intrinsic wave function Ψ_0 which impinges with momentum $\hbar\boldsymbol{k}$ = (0,0, $\hbar\boldsymbol{k}$) on the target in its ground state, described with an intrinsic wave function θ_o . The centre-of-mass wave function is removed from Ψ_0 (θ_o). At the final stage of the reaction, the projectile goes to state a specified by a wave function Ψ_a and the target goes to another state c specified by a wave function θ_c. The state a may be a continuum state that includes some fragments. The momentum transferred from the target to the projectile is $\hbar\boldsymbol{q}$ (Abu-Ibrahim and Suzuki, 2002). Tanihata (1996) defined the scattering amplitude for this reaction written in the Glauber theory as:

$$F_{ac}(q) = \frac{i\boldsymbol{k}}{2\pi}\int db e^{-iq.b}\left\langle \Psi_a\Theta_c \middle| 1 - \prod_{i\in P}\prod_{j\in T}(1-\Gamma_{ij}) \middle| \Psi_o\Theta_o \right\rangle \quad (1)$$

The integrated cross section for this reaction is given by:

$$\sigma_{ac} = \int \frac{dq}{\boldsymbol{k}^2}|F_{ac}(q)|^2, \quad (2)$$

where b is the impact parameter between the projectile and the target.

The profile function Γ in Eqn. (1) is given by:

$$\Gamma_{(b)} = \frac{1-i\alpha}{4\pi\beta}\sigma_{NNe^{-b^2/2\beta}}. \quad (3)$$

The parameters σ_{NN}, α, and β usually depend on either the proton-proton (neutron-neutron) or proton-neutron case. The argument of Γ_{ij} in Eqn. (1) is $\mathbf{b} + \mathbf{s}^P - \mathbf{s}^T$, which stands for the impact parameter between i^{th} and j^{th} nucleons. Here $\mathbf{s}^P$ ($\mathbf{s}^T$) is the two-dimensional coordinates which comprises the x- and y-components of the ith nucleon coordinate in the projectile (target) relative to its Centre-of-mass coordinate.

2.1 Longitudinal Momentum Distribution

At high energies beyond a few hundred MeV/nucleon, one-nucleon removal reaction is contributed by both the elastic and inelastic process with the inelastic process becoming dominant at that energy (Basdevant et al., 2005). After inelastic breakup of the projectile nucleus, the momentum distribution of the core fragment was calculated. Let the momentum of the core be $P = (P_{\perp}, P_{\parallel})$ and that of the nucleon going to the continuum state be $\hbar\mathbf{k}$. Assuming that the core will remain in its ground state, the momentum distribution is calculated by the equation below (Adamu, 2013).

$$\frac{d\sigma_{-N}^{inel}}{dp} = \int \frac{dq}{k^2} \sum_{c\neq 0} \int dk\delta\left(p - \frac{A_C}{A_P}\hbar q + \hbar\boldsymbol{k}\right) |F_{(k,0)c}(q)|^2 \tag{4}$$

Since the momentum transfer received by the ejected valence nucleon will be considered to be large, the final state interaction can be ignored. The continuum scattering wave function of the last nucleon is then approximated by a plane wave:

$$\varphi(r) = \frac{1}{(2\pi)^{2/3}} e^{-p.r}, \tag{5}$$

and Eqn. (4) then becomes:

$$\frac{d\sigma_{-N}^{inel}}{dp} = \int db_N \{1 - e^{-2lm}\chi_{NT}(b_N)\} \times \frac{1}{(2\pi\hbar)^3} \frac{1}{2j+1} \sum_{mm_s} |\int dr e^{\frac{i}{\hbar}p.r} \chi^*_{\frac{1}{2}m_s} e^{i\chi_{CT}(b_N - s)} \varphi_{nljm}(r)|^2, \tag{6}$$

where b_N stands for the impact parameter of the balance nucleon with respect to the target, $\varphi_{nljm(r)}$ is the valence nucleon wave function. The phase-shift functions of the core-target and nucleon-target χ_{CT}, χ_{NT} respectively are defined through the relevant densities (ρ) (Adamu, 2013, Ogawa et al., 2003) as:

$$\int i\chi_{CT}(b) = -\int dr \int dr' \rho_C(r)\rho_T(r')\, \Gamma(\boldsymbol{b} + \boldsymbol{s} - \boldsymbol{s}'), \tag{7}$$

$$\int i\chi_{NT}(b) = -\int dr \rho_T(r)\, \Gamma(\boldsymbol{b} - \boldsymbol{s}). \tag{8}$$

Then the density (ρ) will be given by:

$$\rho_{(r)} = \sum_i c_i e^{-a_i r^2}, \tag{9}$$

and will be normalized to the mass number A of a nucleus as:

$$A = \int \rho(r) dr \quad . \tag{10}$$

Then the integral of Eqn. (6) over transverse momentum leads to the longitudinal momentum distribution (Ogawa et al., 2003).

$$\frac{d\sigma_{-N}^{inel}}{dp_{||}} = \int dp_{\perp} \frac{d\sigma_{-N}^{inel}}{dp} = \frac{1}{2\pi\hbar} \int db_N \left(1 - e^{-2ml}\chi_{NT}(b_N)\right) \int ds(1 - e^{-2ml}\chi_{NT}(b_N - s)) \times$$
$$\int dz \int dz' e^{\frac{i}{\hbar}} P_{||}(z - z') u_{nlj}^{*}(r) \frac{1}{4\pi} P_l(\hat{r}'.\hat{r}), \quad (11)$$

where $r = (s, z)$ and $r' = (s, z')$ and P_l is the Legendre polynomial, U_{nlj} (r) is the single-particle wave function. Then integrating Eqn. (11) over the Legendre polynomial P_l yields σ_{-N}^{inel} (inelastic cross section).

The longitudinal momentum distribution is expressed as the sum of contributions from the azimuthal components of the valence-nucleon wave function (Ogawa et al., 2003). That is:

$$\frac{d\sigma_{-N}^{inel}}{dp_{||}} = \sum_{m_l=-l}^{l} \left(\frac{d\sigma_{-N}^{inel}}{dp_{||}}\right) m_l, \quad (12)$$

with

$$\left(\frac{d\sigma_{-N}^{inel}}{dp_{||}}\right) m_l = \frac{1}{2\pi\hbar} \int db_N \{1 - e^{-2ml}\chi_{NT}(b_N)\} \int ds\{1 - e^{-2ml}\chi_{NT}(b_N - s)\} \times$$
$$\frac{1}{2l+1} \left| \int dz e^{\frac{i}{\hbar}} P_{||}(z) u_{nlj}^{*}(r) Y_{m_l}(\hat{r}) \right|^2. \quad (13)$$

The momentum distribution of the core will come out along the beam direction if $P_{\perp}$=0 in Equation (6), that is:

$$\sum_{m_l=-i}^{i} \frac{d\sigma_{-N}^{inel}}{dP} |_{P_{\perp}=0} = \frac{1}{(2\pi\hbar)^3} \frac{1}{2l+1} \int db_N \left(1 - e^{-2lm}\chi_{NT}(b_N)\right)$$
$$\times \left| \int dr\, e^{\frac{i}{\hbar}} z + i\chi_{CT}(b_N - s)\, u_{nlj} Y_{lm_l}(\hat{r}) \right|^2 \quad (14)$$

3.0 Methodology

The framework of the Glauber model (CSC_GM code) used in calculating cross sections of various reactions for a core plus one valence-nucleon system was modified to enable the calculation of longitudinal momentum distribution of the core fragment after inelastic break-up process (Adamu, 2013). The input data for the calculation of momentum distributions of ^{18}C from the $^{19}C + {}^{9}Be \rightarrow {}^{18}C + {}^{9}Be + n$ reaction system are represented in Table 1A. The input data contained in the file csc.inp specify the reaction system, the momentum distribution, the target and core densities, and the control parameters of the Metropolis algorithm etc. The first line gives the mass numbers of the target, projectile and core (A_T, A_P, and A_C), the second line gives the charge numbers of those nuclei (Z_T, Z_P, and Z_C). The code assumes $A_P - A_C = 1$. The third line defines the incident energy of the projectile per nucleon (in MeV). The fourth line defines the parameters of the nucleon–nucleon profile function, Equation (7): σ_{NN} (in fm^2), $\propto$, and β (in fm^2). For the zero-range profile function, $\beta = 0.0$. The fifth line gives the orbital angular momentum of the valence nucleon. The sixth line specifies the condition for the Monte Carlo quadrature, the number of configuration points (N_s), the step size (δ in fm) for the random walk in the Metropolis algorithm, and the seed number for generating random numbers (irand). The seventh line gives the number of Gaussians used to fit the core and target densities. The eighth line gives the coefficients c_i and the ranges a_i (in fm^{-2}) of the target density. The ninth line gives coefficients c_i and the ranges a_i (in fm^{-2}) of the core density as defined by Equation (9). The last line defines the maximum angle (in degrees) to be calculated. Results of the calculations of the momentum distributions of the core fragment from Equation (14) are written on the file **momdist.out** in Table 2. The multi-dimensional integration over the valence-nucleon coordinates is performed with a Monte Carlo technique. In the Monte Carlo integration, a set of configuration or integration points is generated according to a suitably chosen guiding function *w(**x**)*. The random walk with the Metropolis algorithm is taken to generate such points. The single-particle wave function of the valence nucleon is generated by the code by specifying quantum numbers of the wave function in the input file **wf.inp** in Table 1B**.** To obtain the radial part of the single-particle wave function, $R_{nlj}(r) = ru_{nlj}(r)$ which is used as the guiding function **w(x)** we solve a Schrödinger equation with a potential $U(r)$ (Ogawa et al, 2003):

$$\frac{d^2R(r)}{dr^2} + \frac{2\mu}{\hbar^2}\left[E - U(r) - \frac{l(l+1)\hbar^2}{2\mu r^2}\right]R(r) = 0 \quad (15)$$

$$U(r) = -V_0 f(r) + V_{ls}(l.s){r_0}^2\frac{1}{r}\frac{d}{dr}f(r) + V_{coul} \quad (16)$$

Where

$$f(r) = \left[1 + exp\frac{r-R}{a}\right]^{-1} \quad \text{With } R = r_0 A_C^{1/3}$$

Where r_0 and a are the radius and diffuseness parameters in fm respectively. V_0 is the initial depth of the potential. V_{ls} =17MeV (Adamu, 2013).

4.0 Results And Discussion

The ground state of ^{19}C was assumed to be ^{18}C + neutron system. The valence neutron is in the $1s_{1/2}$ orbit with n = 1, l = 0, j = 0.5, and the separation energy ε= 0.24 MeV (Tanihata, 1996). The longitudinal momentum distribution of the ^{18}C for the reaction $^{19}C+^{9}Be \rightarrow ^{18}C + ^{9}Be + n$ at the energy of 1GeV/nucleon was compared with experiment in Figure 1.

Table 1A. ***csc.inp*** *Input file for* $^{19}C+^{9}Be$ *Reaction System*

S/N	INPUT PARAMETERS	VALUES
1	Mass numbers of target, projectile and core: (A_T; A_P; A_C)	9, 19, 18
2	Atomic numbers of target, projectile and core: (Z_T; Z_P; Z_C)	4, 6, 6
3	Incident Energy per nucleon (in MeV)	1000
4	Profile function parameters (σ_{NN}, α and β in fm^2)	4.320055, -0.2814692, 0.2095773
5	l (angular momentum quantum number)	0
6	Monte Carlo parameters (Ns, δ, irand)	500000, 2.5, −11213
7	Number of Gaussians used to fit the core and target densities	2
8	Coefficient c_i, range a_i (in fm^{-2}) of the Target	-0.06240 0.69377 0.28340 0.30012
9	Coefficient c_i range a_i (in fm^{-2}) of the Core	-1.19721 0.300572 1.35334 0.255236
10	Maximum angle (in degrees)	30

Table 1B. ***wf.inp*** *Input file for* $^{19}C+^{9}Be$ *Reaction System*

INPUT PARAMETERS	VALUES
Initial depth V_0 of the optical potential (in MeV)	70.0
Diffuseness parameter a (in fm)	0.6
Radius parameter r_0 (in fm)	1.2
Energy eigenvalue for the valence nucleon (in MeV)	-0.24
j value for the valence nucleon orbit	0.5
Node number for the valence nucleon orbit	1

Table 2. ***momdist.out*** *output file format*

$P_{\parallel}$[MeV/c]	dσ/dp [mb/(MeV/c)]
0.0000000000000000	2.1716710968696042
10.000000000000000	1.7981598607204563
20.000000000000000	1.0297665377572307
……………………..	………………………

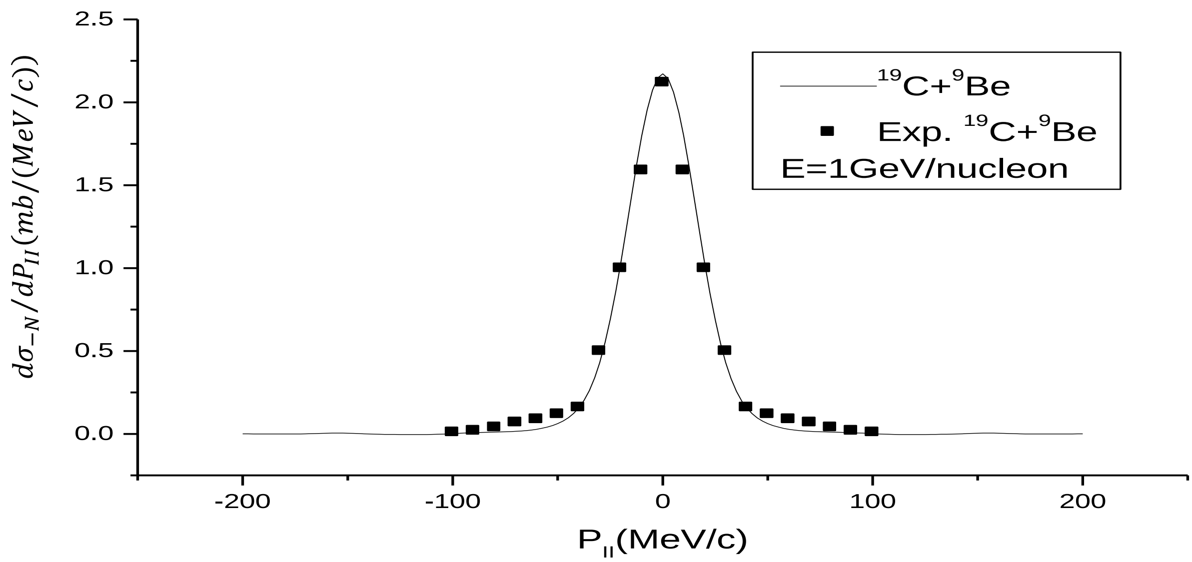

Figure 1. Longitudinal Momentum Distribution of ^{18}C for the reaction $^{19}C+^{9}Be$ at the energy of 1GeV/nucleon. The solid curve denotes the results calculated from Equation (14). The experimental data are taken from *Cortina-Gil et al., (2001).*

4.1 Conclusion

The narrow momentum distributions of the ^{18}C core fragment from $^{19}C+\ ^{9}Be$ reaction induced by the projectile nuclei which has a core plus one valence-neutron suggest a neutron-halo structure for the ^{19}C nuclei. The neutron valence neutron is in the $1s_{1/2}$ orbit with $n = 1$, $l = 0$, j=0.5 and the separation energy of 0.24 MeV (Tanihata, 1996). The results were compared well with the available experimental data.

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